

Andrián Pertout

# La Homa Kanto

for Harmonically Tuned Synthesizer Quartet

No. 393



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No. 393

Composed between August and October, 2005

Hommage à Lou Harrison (1917-2003)

'Acousmatic' Premier: 27 April, 2007 (St. Neighborhood Unitarian Church, Los Angeles Sonic Odyssey: Electronic and Computer Music Concert Series 2007, 26-28 April, 2007, Pasadena, California, USA)

Duration: 11'50"

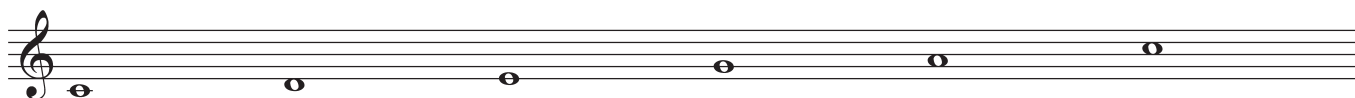
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## PROGRAMME NOTES

In order to understand the basic concept of just intonation, one must refer to the natural acoustic phenomenon known as the 'harmonic series'. The overtones of a specific pitch are generally referred to as the 'harmonic series', and the musical scale derived from this series is constructed around pure (or just) intervals. This system of just intonation is strikingly dissimilar to the 'twelve-tone equally-tempered division of the octave', which is based on the division of the octave into twelve equal intervals – the frequency ratio of each semitone (or tempered half-tone) being the twelfth root of 2. In *Lou Harrison: Composing a World* Leta E. Miller and Fredric Lieberman describes just intonation thus: "Pure intervals arise when the frequencies of the individual tones reflect the precise mathematical proportions that occur in the series: 3/2 for the fifth, 4/3 for the fourth, etc." Intervals manifested naturally within the harmonic series are particularly favourable in just intonation, and certainly ones with "superparticular vibration ratios", where the "numerator exceeds the denominator by one" such as the just perfect fifth (3/2), perfect fourth (4/3), major third (5/4) and minor third (6/5).

'La Homa Kanto' or 'The Human Song' in Esperanto is a dedication to the late American composer Lou Harrison (1917-2003), and its pitch material has been derived directly from Harrison's five-tone scales, presented in *Lou Harrison's Music Primer: Various Items About Music to 1970*. Included are the first five in the series, with the first – the diatonic or major pentatonic scale – acknowledged by Harrison as the "prime pentatonic", and "practically the Human Song." According to Harrison, "These first five are the most widespread, the core, the principal modes of 'Human Music'. They also constitute the bone-work, the firmest compositional basis for seven-tone music." Modes one, two and five may be considered to be anhemitonic pentatonic forms (scales incorporating half steps, or semitones), while three and four, hemitonic pentatonic forms (scales not incorporating half steps, or semitones). Harrison's five pentatonic scales include the diatonic or major pentatonic, minor pentatonic (or fifth mode major pentatonic), Japanese *Hirajoshi*, fifth mode Indonesian *Pélog*, as well as the Indonesian *Sléndro* (or second mode major pentatonic). The consideration of their complements presents the third mode major pentatonic, fourth mode major pentatonic, third mode Indonesian *Pélog*, fourth mode Japanese *Hirajoshi*, as well as the Indonesian *Sléndro* (or second mode major pentatonic).

### Major Pentatonic Scale "The Human Song"



The compositional strategy for the work involves the creation of two complimentary sets of melodic material, consisting of what is essentially a four-bar melodic sentence based on Harrison's first pentatonic (or major pentatonic), with an additional four-bar variant based on its complement (or third mode major pentatonic). In some aspects, this second sentence could be considered an almost comparable inversion, although it does not represent an 'authentic' inversion in the serial sense of the word, but rather an inversion of contour utilizing the inverted pitch material of the original major pentatonic. The third bar of both versions represents the only literal inversion of intervals. A further observation reveals the respective major and minor tonalities of the two four-bar melodic sentences.

#### Four-bar melodic sentence



#### Complimentary sentence



The work also features ten distinct tuning modulations: three-limit, five-limit, seven-limit, eleven-limit, thirteen-limit, seventeen-limit, nineteen-limit, twenty-three-limit, twenty-nine-limit, and thirty-one-limit just intonation systems, based on the third, fifth, seventh, eleventh, thirteenth, seventeenth, nineteenth, twenty-third, twenty-ninth, and thirty-first partials of the harmonic series – each system adding its own microtonal nuances to the recurring melodic material, which is further transformed via the introduction of alternative scalar material, as well as via harmonic development pertinent to each individual just intonation system. The structural framework of the composition is directly related to the primary motive of the four-bar melodic sentence. In five-limit terms, the primary motive denotes the following intervals:  $5/4$ ,  $3/2$ , and  $9/8$ , and  $5/4$ , and  $3/2$ ; and is expressed in the score as section one (rehearsal letters A and B – incorporating Lou Harrison Pentatonic #1, and both three-limit and five-limit just intonation), which is designated a  $5/4$  metrical structure; section 2 (rehearsal letters C and D – incorporating Lou Harrison Pentatonic #2, and both seven-limit and eleven-limit just intonation), a  $3/2$  metrical structure; section 3 (rehearsal letters E and F – incorporating Lou Harrison Pentatonic #3, and both thirteen-limit and seventeen-limit just intonation), a  $9/8$  metrical structure; section 4 (rehearsal letters G and H – incorporating Lou Harrison Pentatonic #4, and both nineteen-limit and twenty-three-limit just intonation), a  $5/4$  metrical structure; and section 5 (rehearsal letters I and J – incorporating Lou Harrison Pentatonic #5, and both twenty-nine-limit and thirty-one-limit just intonation), a  $12/8$  metrical structure (equal to  $3/2$ ). A sequential series of metrical structures is therefore utilized in the work to reflect the primary motive and its frequency ratios of  $5/4$ ,  $3/2$ ,  $9/8$ ,  $5/4$ , and  $3/2$ .

Saturday, 3 February, 2001 (Our Colleague Lou Harrison)

Dear Fellow SCI Members:

"I just received very sad news from my colleague and fellow SCI member Donald Harris. He reports that Lou Harrison, noted American composer, passed away last night.

"Lou Harrison had appeared at several of our National Conferences. His cheerful, luminous personality endeared him to and will be remembered by all who knew him. Lou was en route to Ohio State for our annual New Music Festival which honours him and features his music.

"I include the message I received from Don Harris, who related the sad news to our faculty."

*Thomas Wells (President, Society of Composers, Inc.), Professor of Music, Director, Sound Synthesis Studios, Ohio State University, Columbus, Ohio, USA*

Saturday, 3 February, 2001 (Our Colleague Lou Harrison)

"Last night, while travelling from Chicago to Columbus, Lou Harrison passed away. We had sent Adam Schweigert and Joe Panzner, two students in Composition and Theory, to Chicago with a University van to greet Lou and his travelling companion, Todd Burlingame, and drive them back to campus for the Festival. Lou does not like to fly and took the train, the California Zephyr, from near San Francisco to Chicago. The train arrived at about 5:15 PM. While en route to Indianapolis, their overnight destination, Adam, Joe, Lou and Todd decided to stop at a Denny's restaurant in Lafayette, Indiana, for some dinner. Lou stumbled and fell upon getting out of the vehicle and apparently suffered a heart attack. He had great difficulty breathing. Paramedics were called and they arrived within minutes. Lou was transported to the hospital but was unable to be revived. He passed away around 9 or 9:30 PM.

"This is a great tragedy for the entire world of music. Lou was as excited about the Festival as we were. Our sympathies go out to all in the world of music and dance who treasured his great gift. I will always remember the joy in his voice when we spoke over the telephone about the Festival. He was an active participant in planning the concerts. He had also prepared a special seminar for OSU students in composition. He was very much looking forward to the CSO performances of his Third Symphony in a final revision that he had just completed. He told me how honoured he felt to have so many of his compositions performed on the Festival. Let everyone know that the Festival performances will go on as planned as a tribute to this legendary American musician.

"I must also at this time offer our sincere thanks to Adam and Joe. As you might imagine, they were devastated by what took place. Yet they remained calm at all times, stayed in close touch with me, and were of great comfort to Todd Burlingame. We can be proud of the way they represented Ohio State and the School of Music. Adam and Joe will return to Columbus today. When you see them, please thank them for their calm and measured reactions under such tragic circumstances.

"Lou Harrison will be cremated today; his remains shipped back to California. He was eighty-five years old."

*Donald Harris, Professor of Composition and Theory, School of Music, Ohio State University, Columbus, Ohio, USA*

Thursday, 6 February, 2003 (Death of Lou Harrison)

Dear Thomas Wells,

“Thank you very much for sharing the sad information about Lou Harrison. And what a beautiful tribute by long-time SCI member, Donald Harris. Lou attended symposia in his honour many times here in Portland, Oregon, Lou's place of birth. Two recent ones in particular stand out when Lou was aged 79 and 82. The first was at Lewis & Clark College in Portland with Bill Colvig, his late partner. Lou gave numerous inspiring lecture presentations that took your breath away, and when his music was performed by L&C students to a large public audience, Lou signed CD's late into the night despite his need to get up early the next morning to present again and again for several days. His music was, and continues to be, performed often in Portland on many of the instruments Lou invented with Bill Colvig.

“Do you remember how Lou would title many of his musical compositions in Esperanto?! Lou was passionate about keeping this rarely used universal language alive. Lou also designed numerous letter fonts, many of which we now select on the internet when writing our documents. Remember his beautiful calligraphic handwriting and the painstaking care with which he wrote each letter?

“At aged 82, Lou again came to Oregon and was honoured in a Symposium at the University of Oregon in Eugene where he taught a roomful of young composers for several days. Lou was a lively and brilliant presenter at these events. His eyes and whole body lit up when he spoke, and his ideas were often more controversial than those of students 60 years younger! Despite his exceptional brilliance, he never boasted of his intelligence or of anything else, instead inspiring the listener to invent and create as he did, living life to the fullest. Even into his 80's, the cogency of his ideas, his wit, passion, and joy of learning were as strong as that of a young person. Your choice of the word ‘luminous’ to describe Lou is perfect. He was a wonderful, brilliant human being, and we will all miss him greatly. I can only imagine what your New Music Festival is like with the aura of Lou Harrison hovering in the room. The music must be exceptional and every word spoken charged with power. I'm sure Lou is there with you and I wish I could be there too.

“Warm remembrances, and thank you for writing.”

*Linda Hathaway Bunza, Director, Columbia Research Institute for the Arts and Humanities, Portland, Oregon, USA.*



## Lou Harrison, Beloved Icon and Pioneer

By Fredric Lieberman

The musical world was stunned by the death, on Sunday, February 2, of Lou Harrison, the distinguished and much beloved composer and humanitarian. In his delightful *Music Primer* (1971) Lou wrote, 'Someone has said that music is to be recommended because the Angels practice it.' Self-proclaimed a 'glandular optimist,' certainly Lou is now cherishing the songs of that Angelic choir, and I imagine he's already started teaching them to sing in Just Intonation.

Harrison, who celebrated his 85th birthday this year, was on his way to a week-long festival of his music in Columbus, Ohio, when an accidental fall apparently brought on a heart attack. The performances will go on as a memorial tribute. Lou had recently completed revisions to his *Third Symphony* – he continually revised works until they 'stopped itching' – and was enthusiastically looking forward to the premiere of the new version next Friday. (The *Third Symphony* was first performed in 1982, though its diverse movements include material from as early as 1937.)

Lou met his life-partner, William Colvig, in 1967; they lived and worked together until Bill's death in 2000. Lou's younger brother Bill also predeceased him. He is survived by his sister-in-law Dorothy, of Reno, Nevada, and two nephews Brad and Barry.

Though beset by bouts of illness, sometimes quite painful, Lou never stopped working, teaching, and learning. He recently completed a new book *Poems and Pieces*, currently in press, which juxtaposes poetry with a selection of works for the Javanese gamelan. In addition to revising the *Third Symphony*, Lou completed revisions and additions to his second opera *Young Caesar*; completed the construction of an ecologically-friendly straw-bale house of his own design in Joshua Tree, California; bought a used Mercedes diesel auto, which he had converted to run on bio-diesel fuel (that is, any form of vegetable oil); and was preparing once again to teach his course on Javanese gamelan for Cabrillo College in Santa Cruz.

As pianist Sarah Cahill pointed out, 'Lou's passing is so symbolic of the end of an era. His deep connections to Schoenberg, Ives, Cowell, and Cage made him a real icon, even beyond his own compositional career.' Born in Portland, Oregon, in 1917, Lou grew up in the San Francisco Bay area, studied music at San Francisco State College, composition with Henry Cowell and Arnold Schoenberg, and pioneered with John Cage in creating and performing works for percussion ensemble. Moving to New York in 1943, Harrison worked as a music critic for the Herald Tribune, hired by Virgil Thomson, and wrote also for *Modern Music* and *New Music Quarterly*.

Lou had previously corresponded with Charles Ives, and during this New York period conducted the premiere of Ives's *Third Symphony*, which won the Pulitzer Prize. Ives wrote to Lou 'As you are very much to blame for getting me into that Pulitzer Prize street, and for having a bushel of letters to answer and for having a check of \$500 thrown over by the Trustees of Colum-Uni-, you have got to help me by taking 1/2 of this (somewhere enclosed)...'

### **Fusing Asian with Western music**

Lou returned to California in 1953, where he settled in Aptos, near Santa Cruz. Life in California, though difficult for a struggling composer, brought him into close contact with Asian musics, which inspired melodic and modal experiments, leading to a panoply of works fusing Asian with Western music in unique and beautiful ways. Some of these works are composed directly for traditional Asian instruments, such as the many compositions for Javanese gamelan; others combine Asian and Western instruments, one of the most successful being *Pacifika Rondo* (1963); and still others, such as the magnificent *Piano Concerto* (1985) borrow musical forms or styles from Asian sources, so thoroughly integrated into the musical texture that their diverse origins are buried beneath the surface of Lou's melodic magic. Harrison's major works include four symphonies, two operas, the *Piano Concerto*, major dance scores including *Solstice* (for Jean Erdman) and *Rhymes With Silver* (for Mark Morris), the *Strict Songs* which, when recorded by the Louisville Symphony, introduced many listeners to pure intonation, and many chamber works of great power and beauty, such as the *Suite for Piano*, the *Grand Duo* for vn and pf, the *Varied Trio* for vn, pf, and percussion, and the *String Quartet Set*. These works will, I'm sure, stand the test of time.

Harrison pioneered in many fields: music for percussion orchestra; the use of pure or Just Intonation rather than equal temperament; the creation of new instruments; the introduction of gamelan music and new composition for that ensemble; and teaching world music, which he did for decades at Mills College, San Jose State, and Cabrillo College.

Lou was a founding spirit of the Cabrillo Music Festival, which has grown to be one of the most important venues for new music. In 2001, under the baton of Marin Alsop, the Cabrillo Festival performed Lou's first opera, *Rapunzel* that had been premiered at the 92nd St. Y in New York City in 1959; over the years other conductors including Gerhard Samuel, Carlos Chavez, and Dennis Russell Davies have led the Cabrillo Festival orchestra in presenting Harrison's music, along with that of hundreds of other contemporary composers.

### **A fearless pacifist, outspoken environmentalist**

On social issues Lou was fearless and outspoken. A pacifist, he protested both in words and music against atomic testing, the Vietnam War, and militarism in general. He was an early supporter of environmental causes, championing the use of renewable fuels, paper recycled, or even better made from fast-growing, easily renewable hemp or kenaf. An advocate for Gay Rights, Lou never hesitated to make his views public. In 1963, at the University of Hawaii, Lou fired off a letter to the student paper, protesting one previously published: 'In the present population explosion, calling homosexuals sick strikes this reader as a case of pots and kettles.'

Mr. Ricardo Trimillos, then a graduate student, recalls: 'Lou was not only a wonderful artist; he was a passionate and committed citizen. I remember his letter defending gay rights back in '63 when being gay was hardly on anyone's radar screen. He signed it as 'Composer in Residence' for the contemporary festival, which upset some faculty members. However, for me he brought home the 'ideology' of understanding music in its cultural context in a very dramatic way. For me that was an important lesson, not taught, but learned.'

Harrison left a voluminous musical legacy of over 300 compositions. Over the past decade, his work has been rapidly growing in international acclaim. New CDs appear regularly; festivals and performances have been increasingly abundant. As of this writing, a memorial tribute in Santa Cruz is being organized, and others will follow.

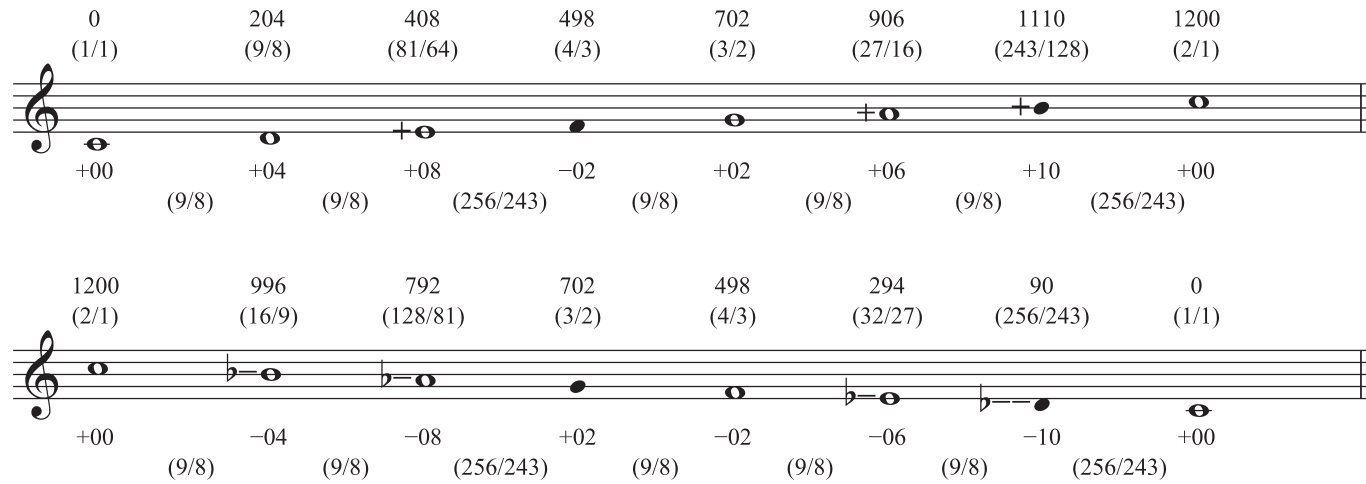
### **Contagious enthusiasm, generosity, and love**

For me, Lou inspired not only through his music, with its continuous search for beauty heedless of current styles or fads, but also through his contagious enthusiasm, generosity, and love. Seldom would I come away from a visit to Lou's Aptos home without the exciting feeling of having learned something remarkable, whether about music, architecture, politics, or literature. Lou filled his place in the world fully and energetically – it's hard to imagine what the world will feel like without him.

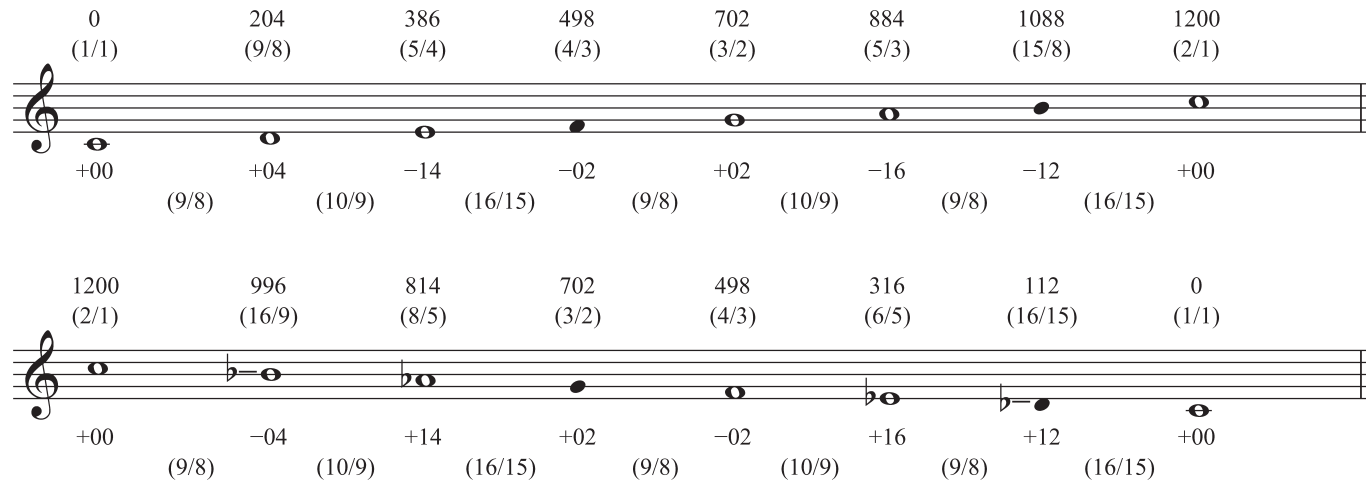
(Fredric Lieberman, composer and ethnomusicologist, teaches at the University of California, Santa Cruz. With co-author Leta Miller, he has written *Lou Harrison: Composing a World* (1998) and a second book on Lou Harrison is forthcoming from the U. of Illinois.) ©2003 Fredric Lieberman, all rights reserved.

*San Francisco Classical Voice, Oakland, California, USA*

### Lou Harrison Pentatonic #1 3-limit Intonation



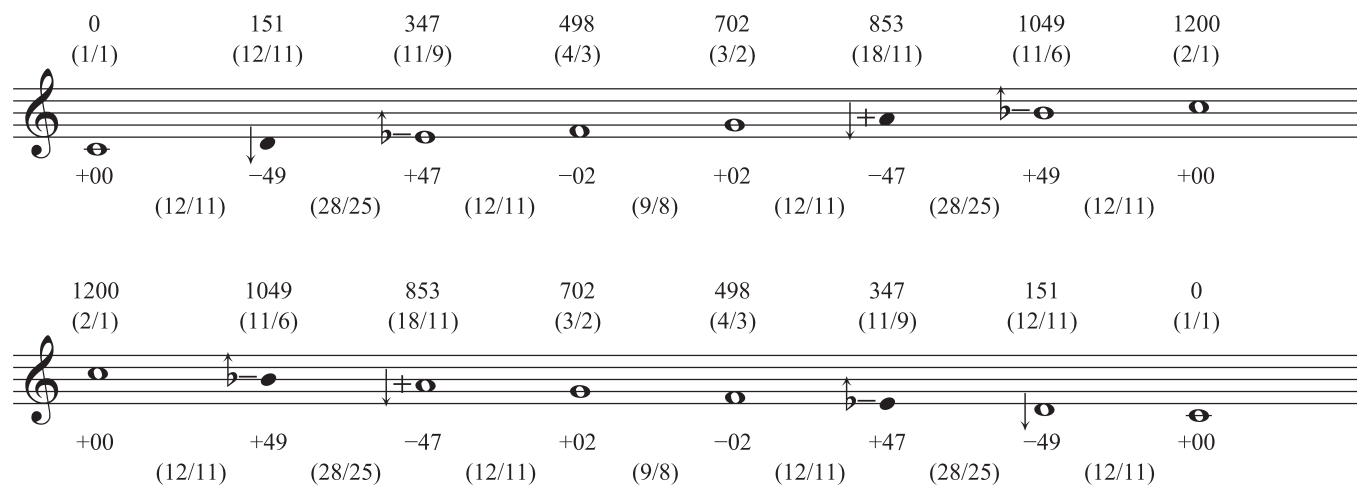
### Lou Harrison Pentatonic #1 5-limit Intonation



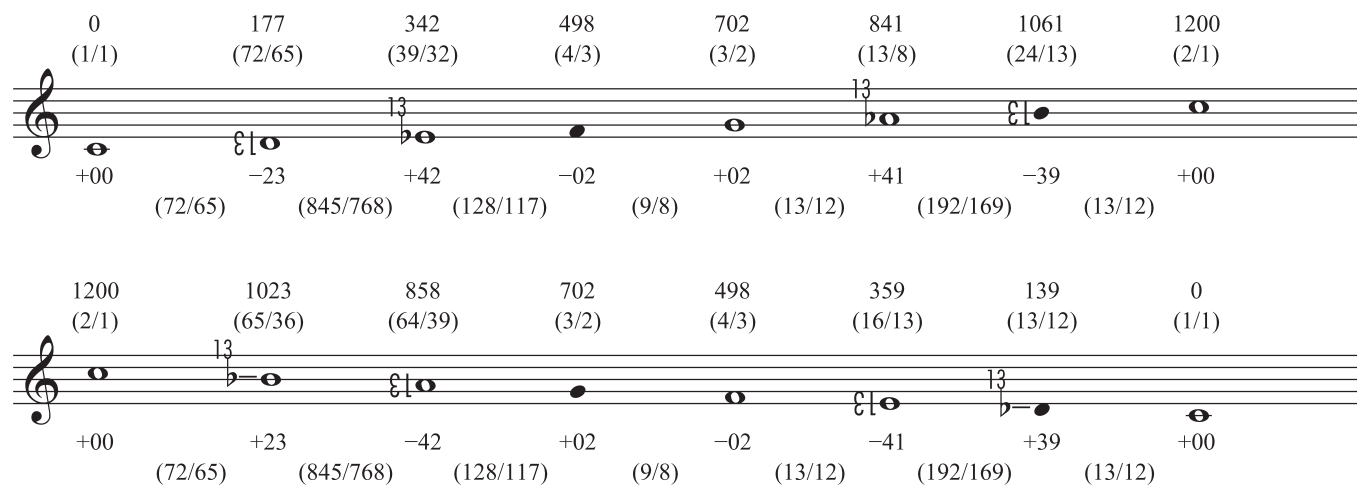
# Lou Harrison Pentatonic #2 7-limit Intonation



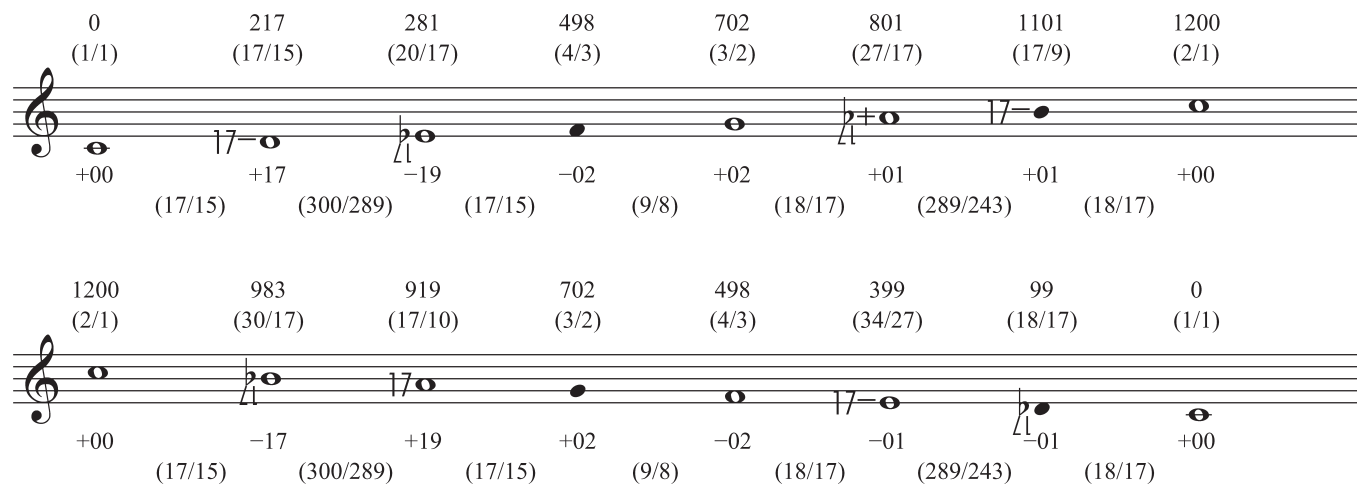
# Lou Harrison Pentatonic #2 11-limit Intonation



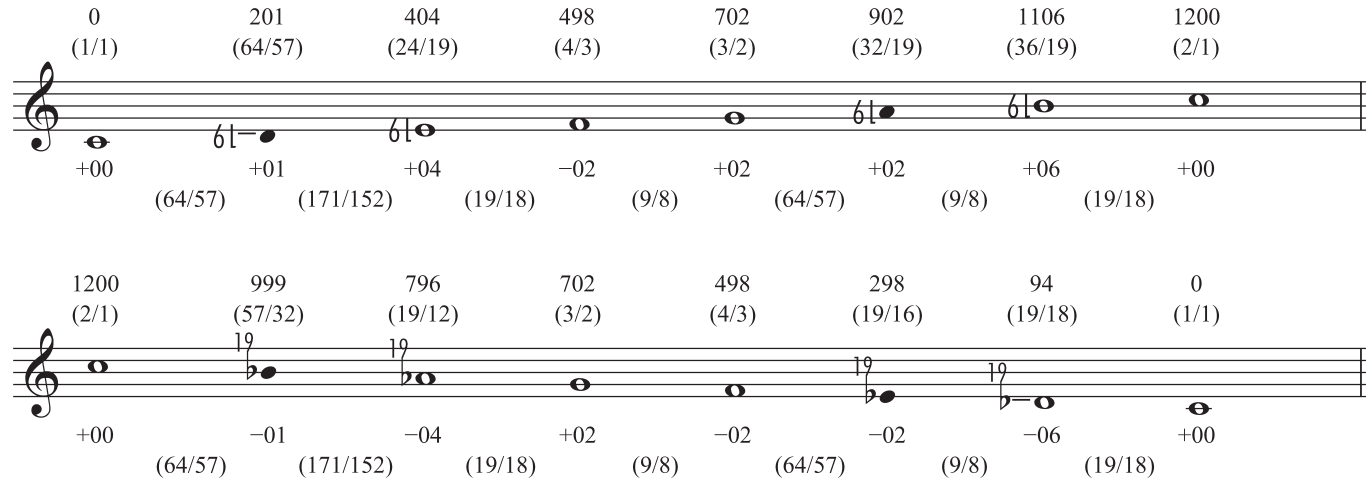
### Lou Harrison Pentatonic #3 13-limit Intonation



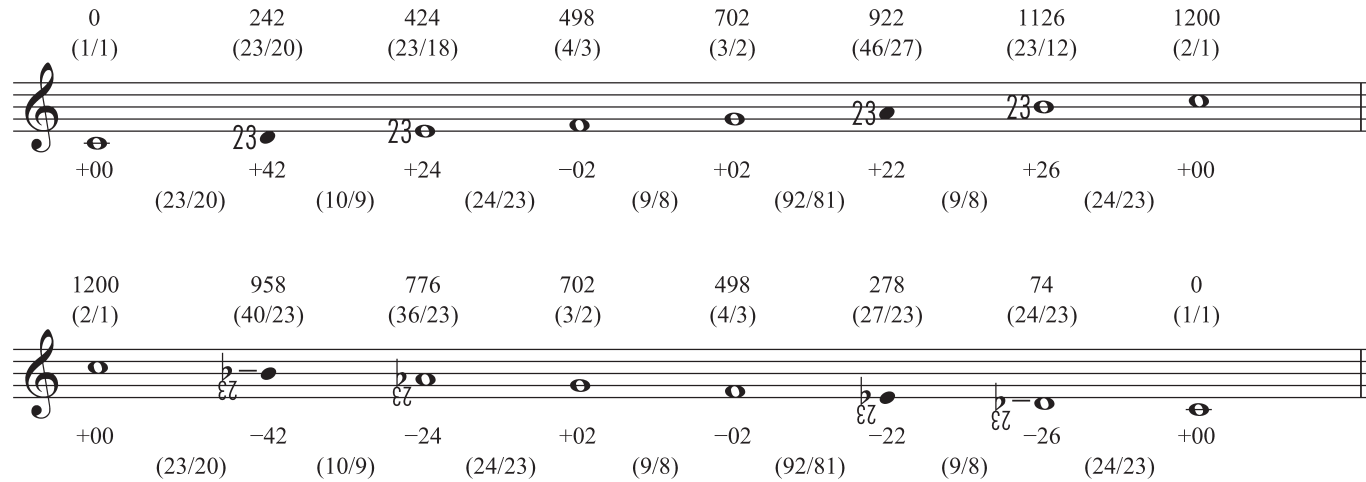
### Lou Harrison Pentatonic #3 17-limit Intonation



Lou Harrison Pentatonic #4  
19-limit Intonation



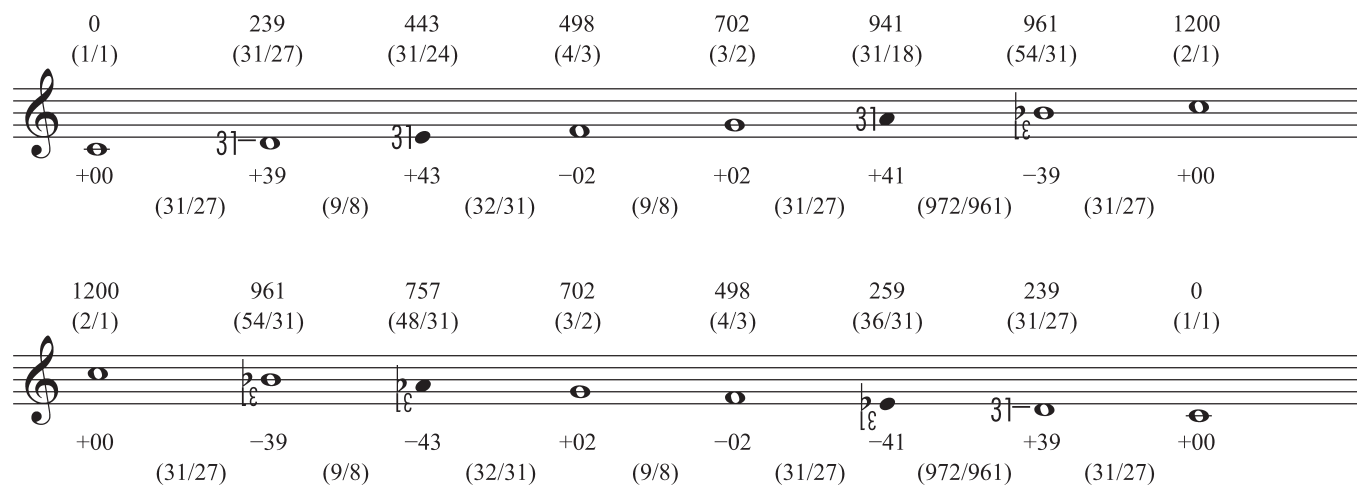
Lou Harrison Pentatonic #4  
23-limit Intonation



# Lou Harrison Pentatonic #5 29-limit Intonation

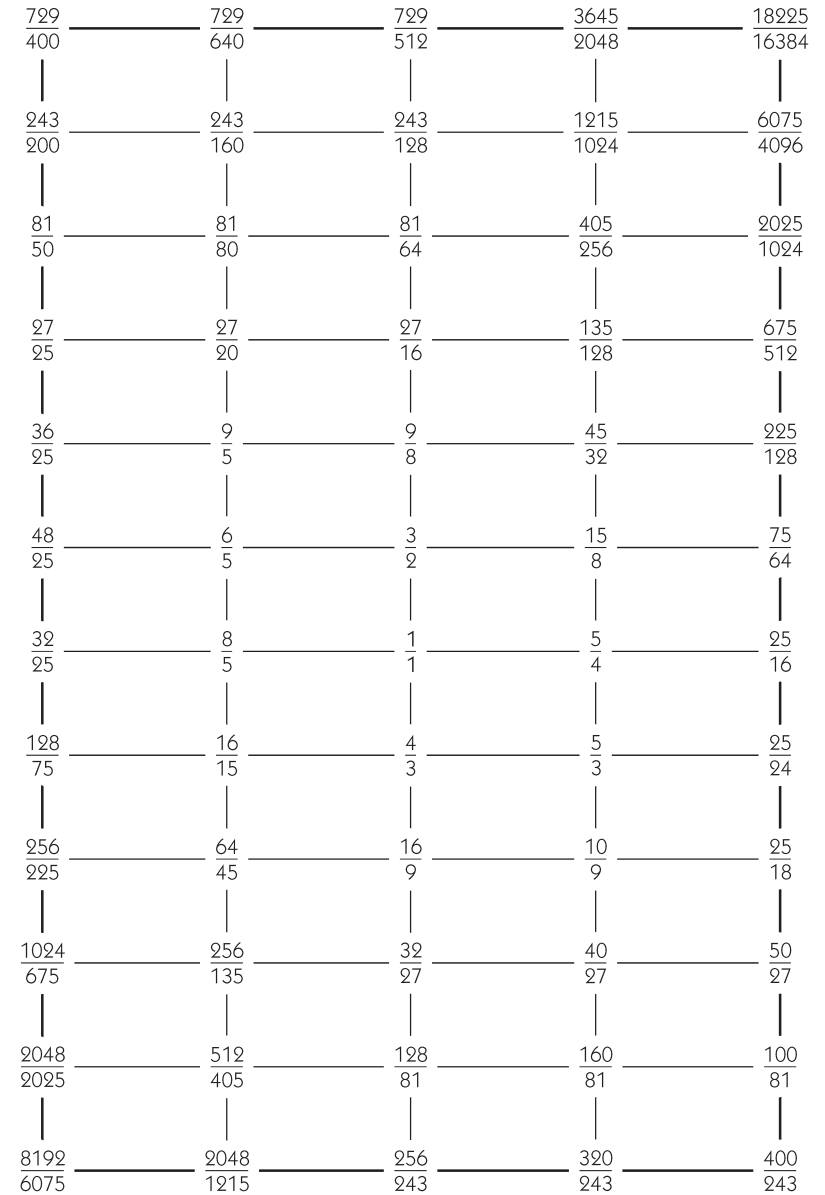
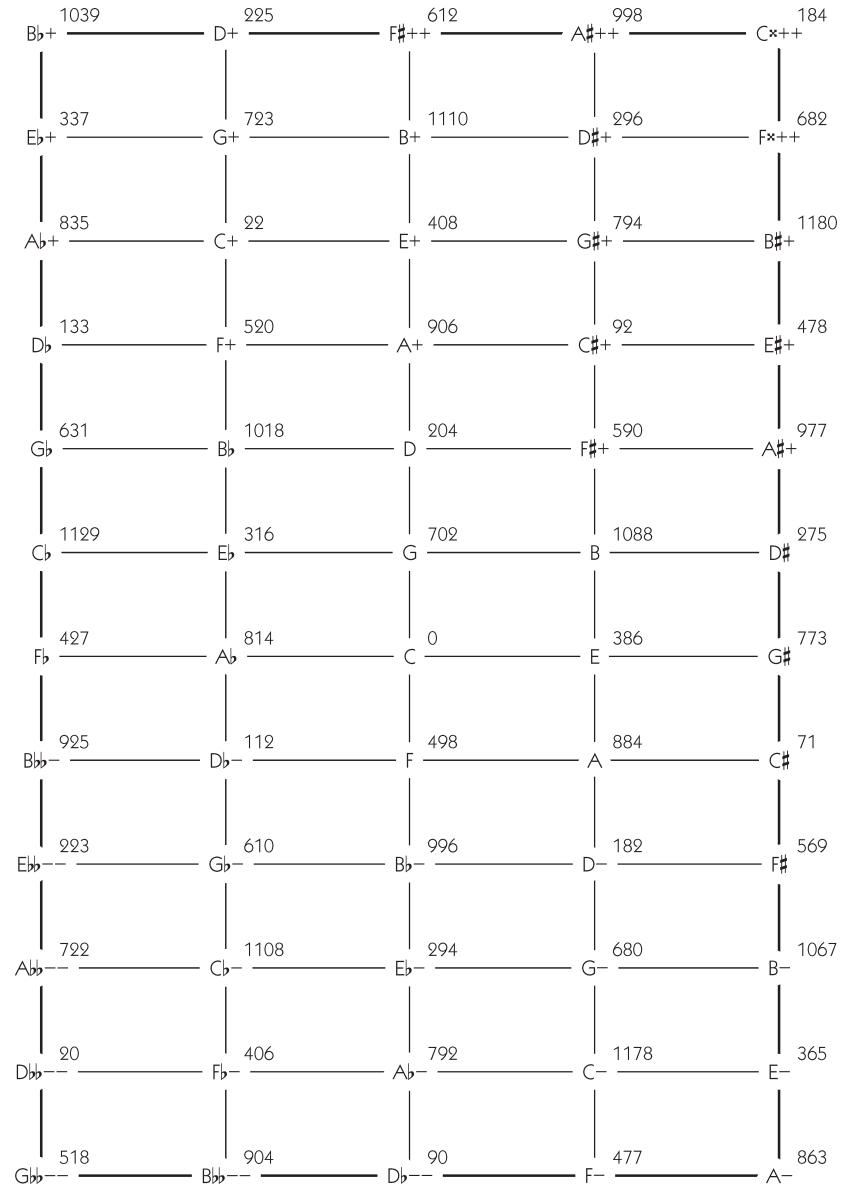


# Lou Harrison Pentatonic #5 31-limit Intonation

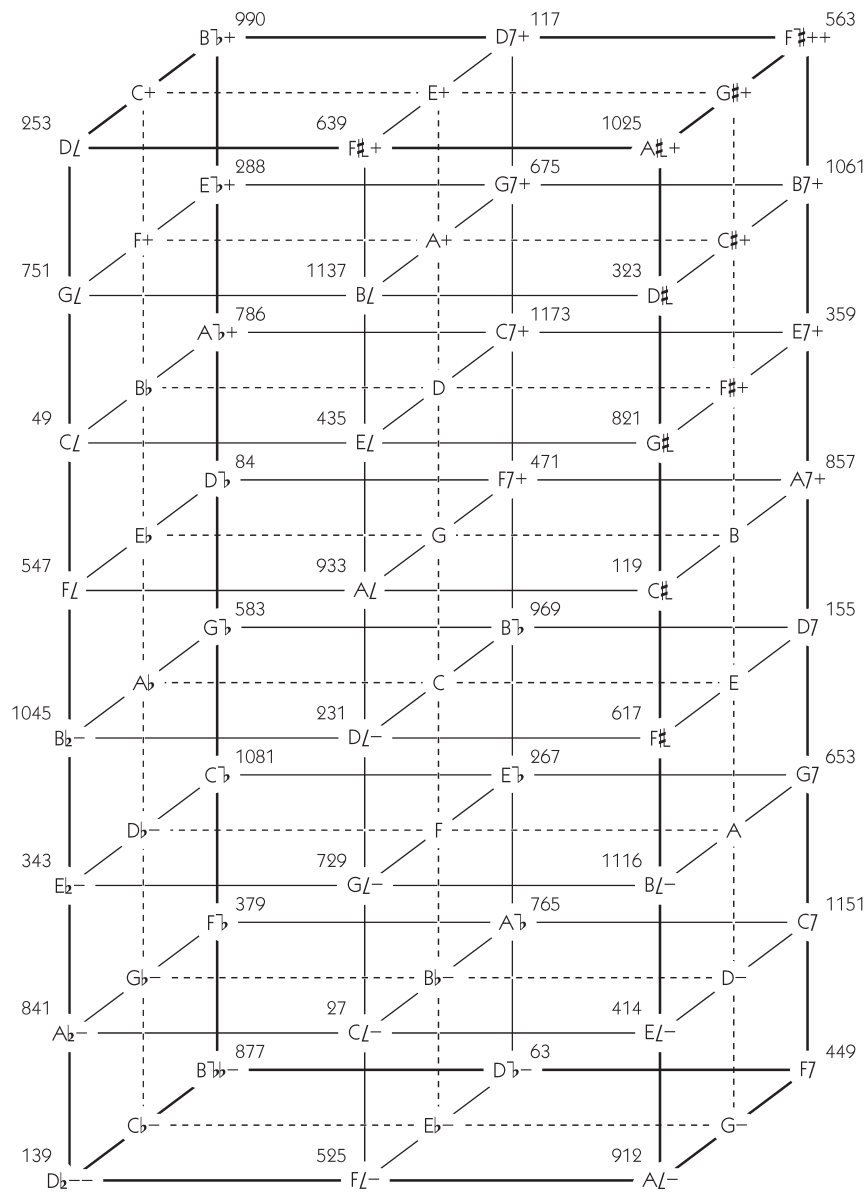




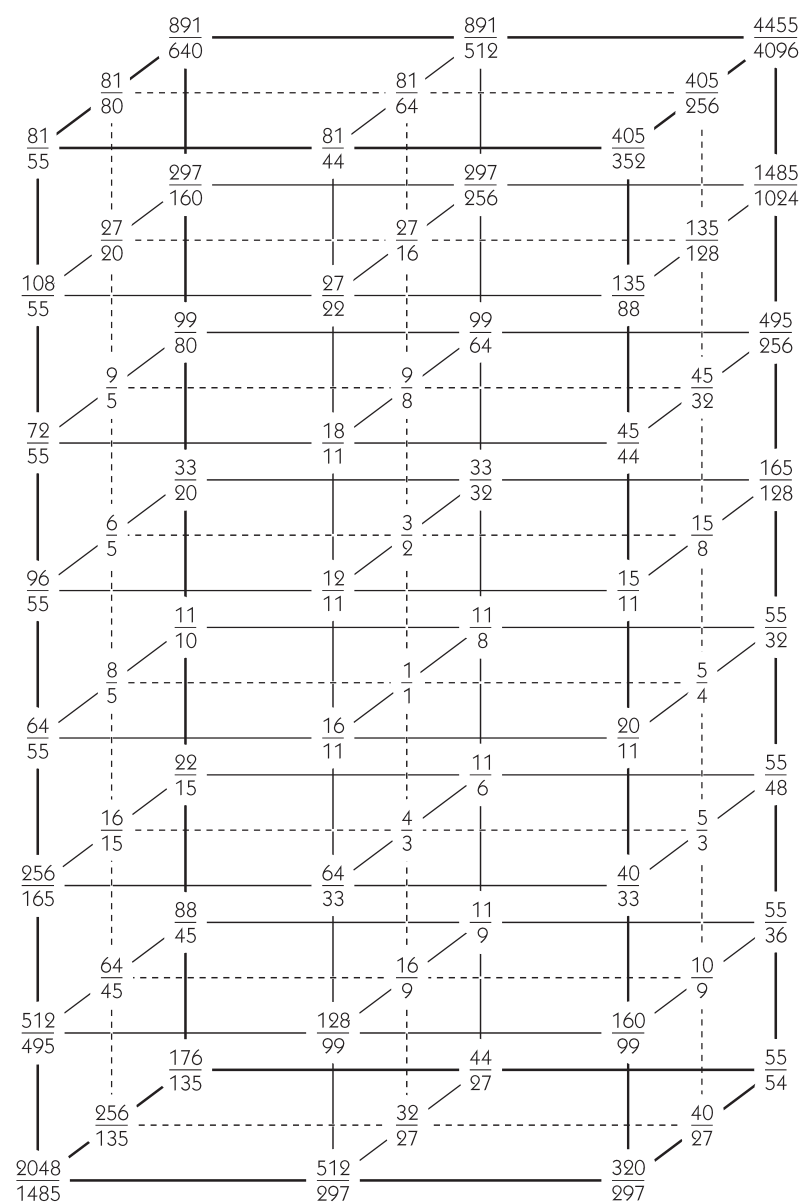
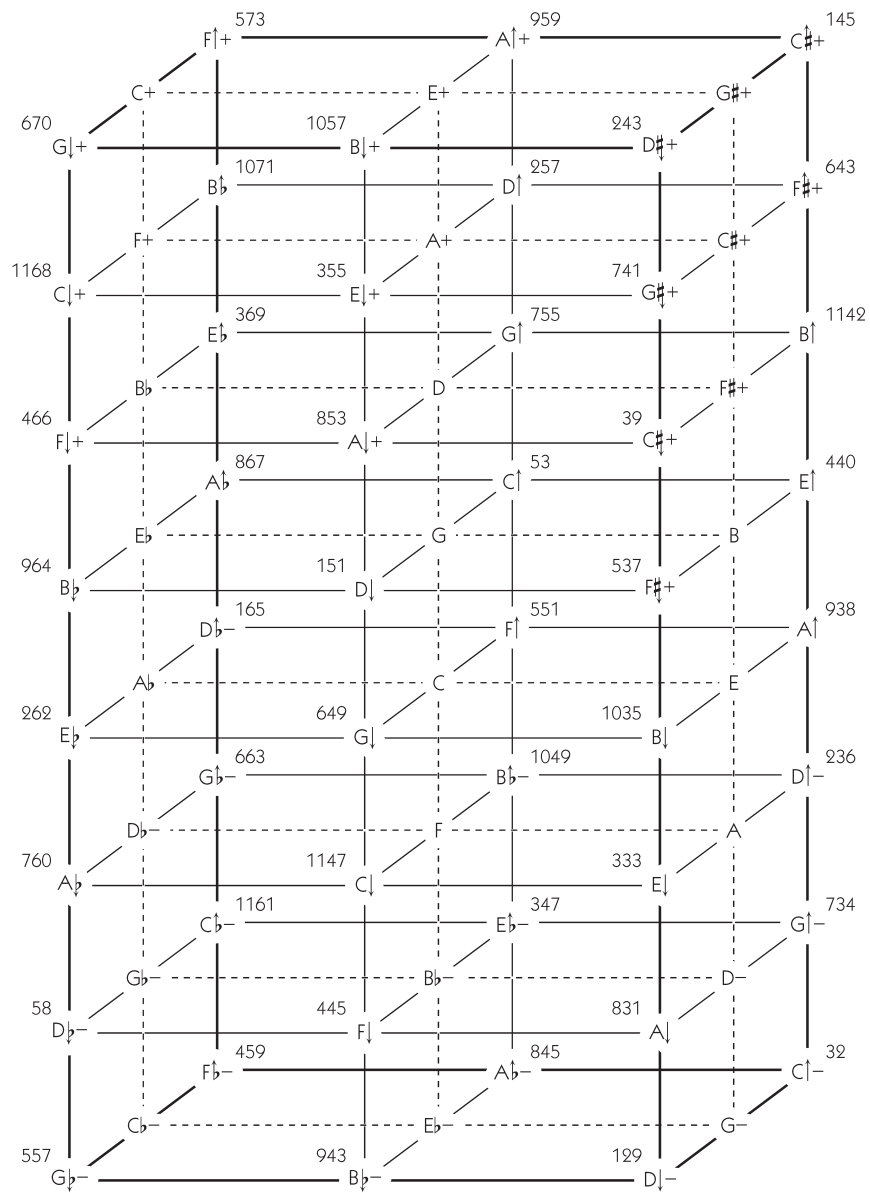
## 2, 3, 5-limit Pitch Lattice (Notation, Ratio and Cent Values)



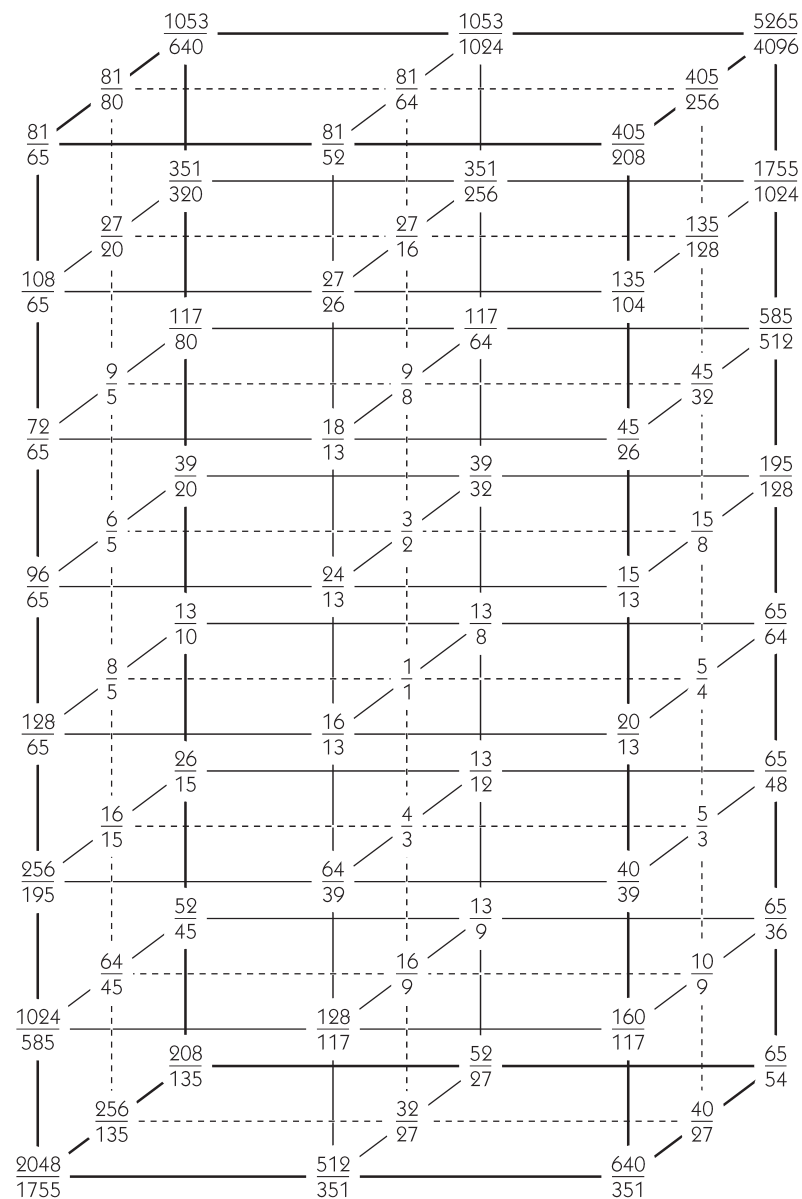
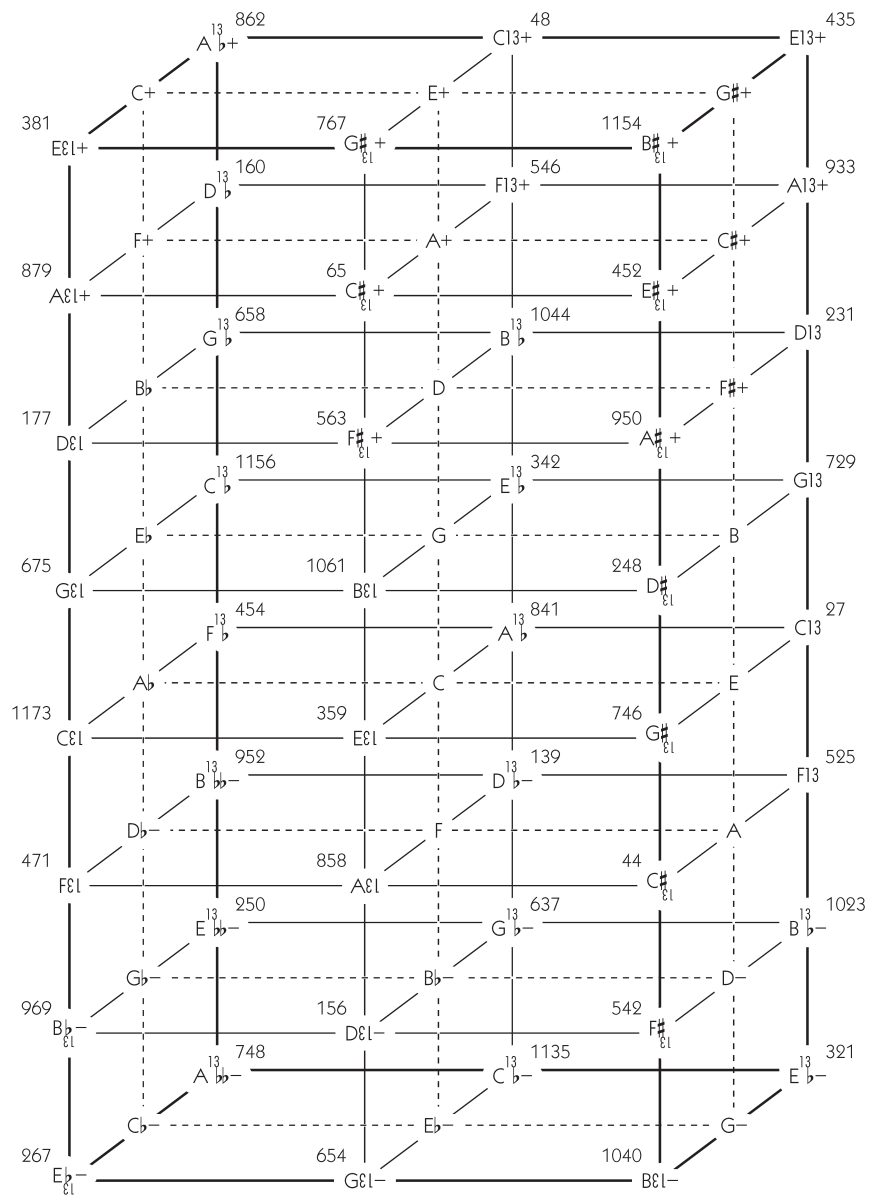
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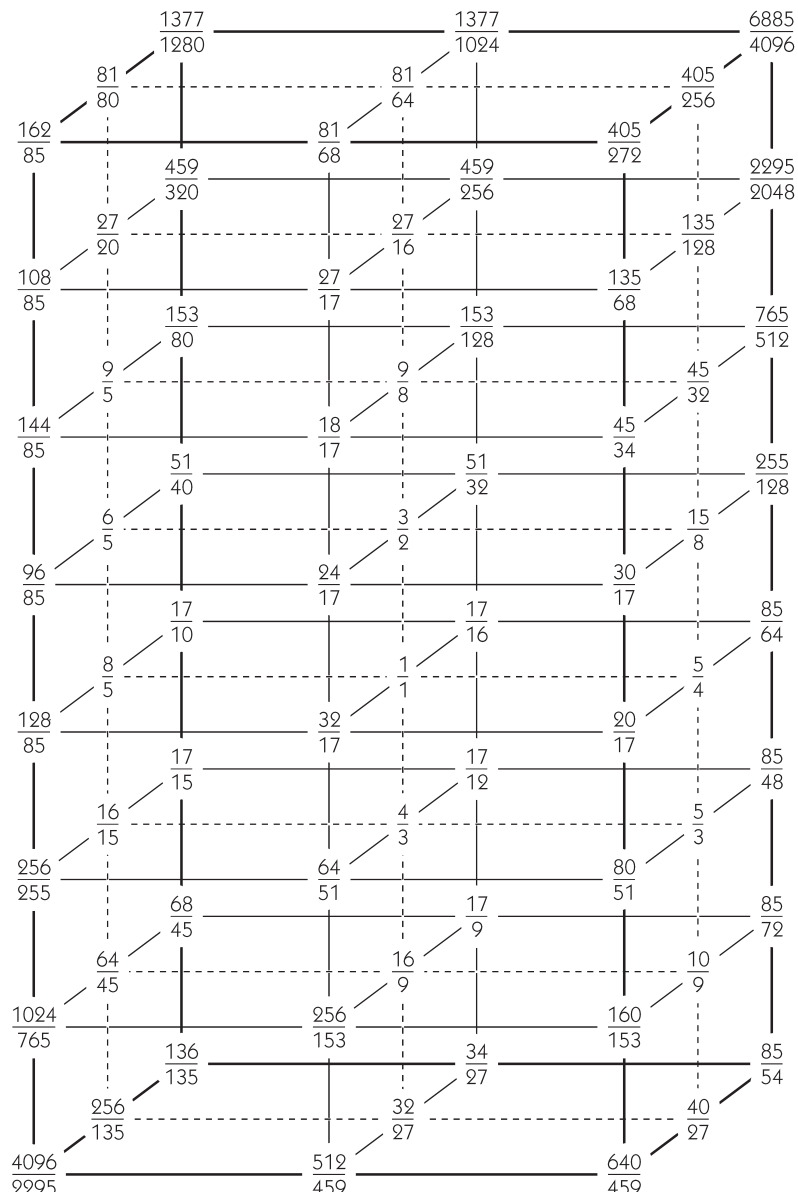
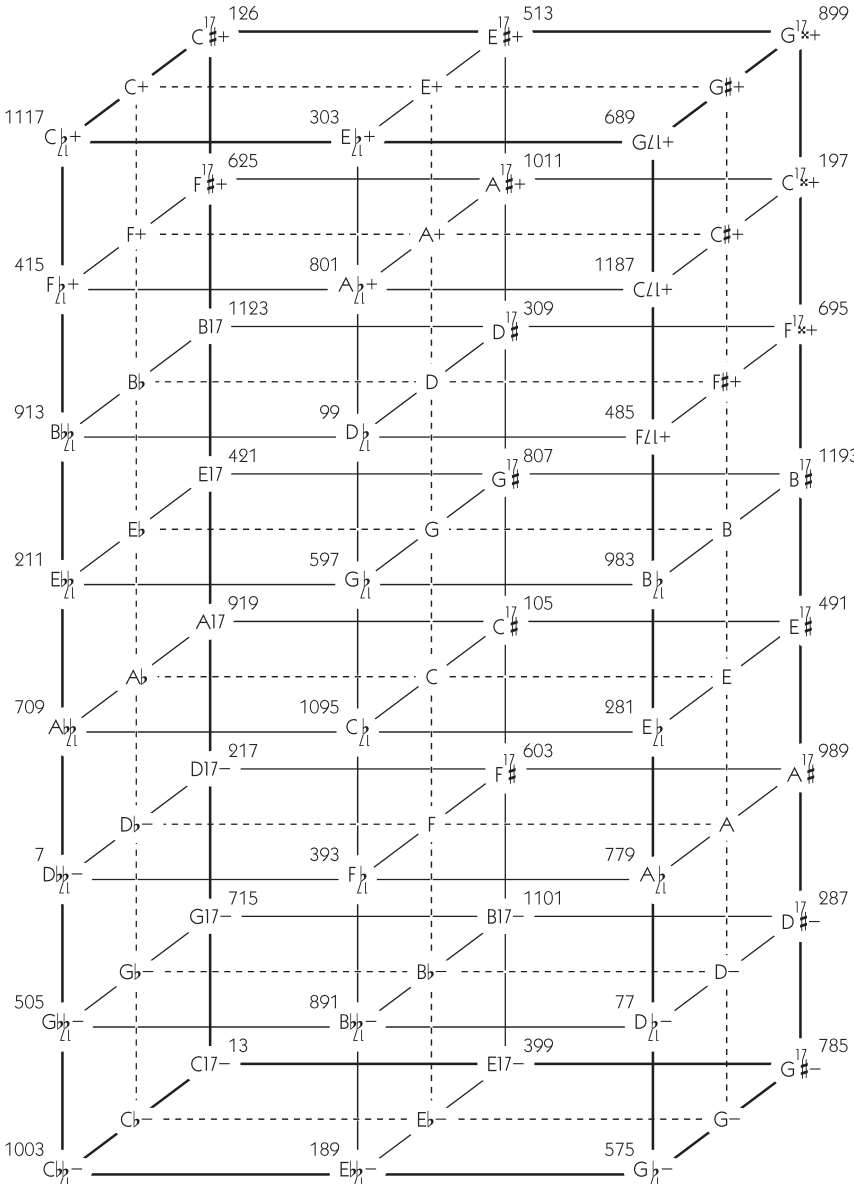
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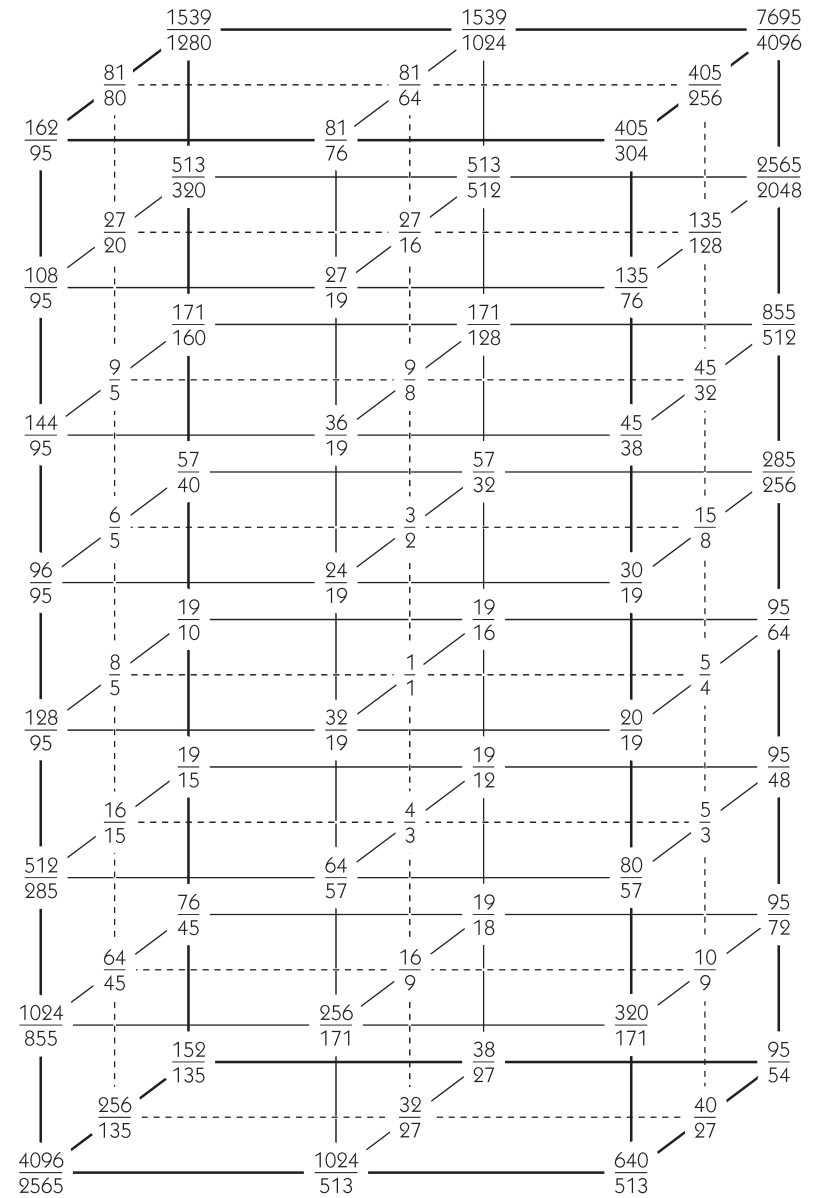
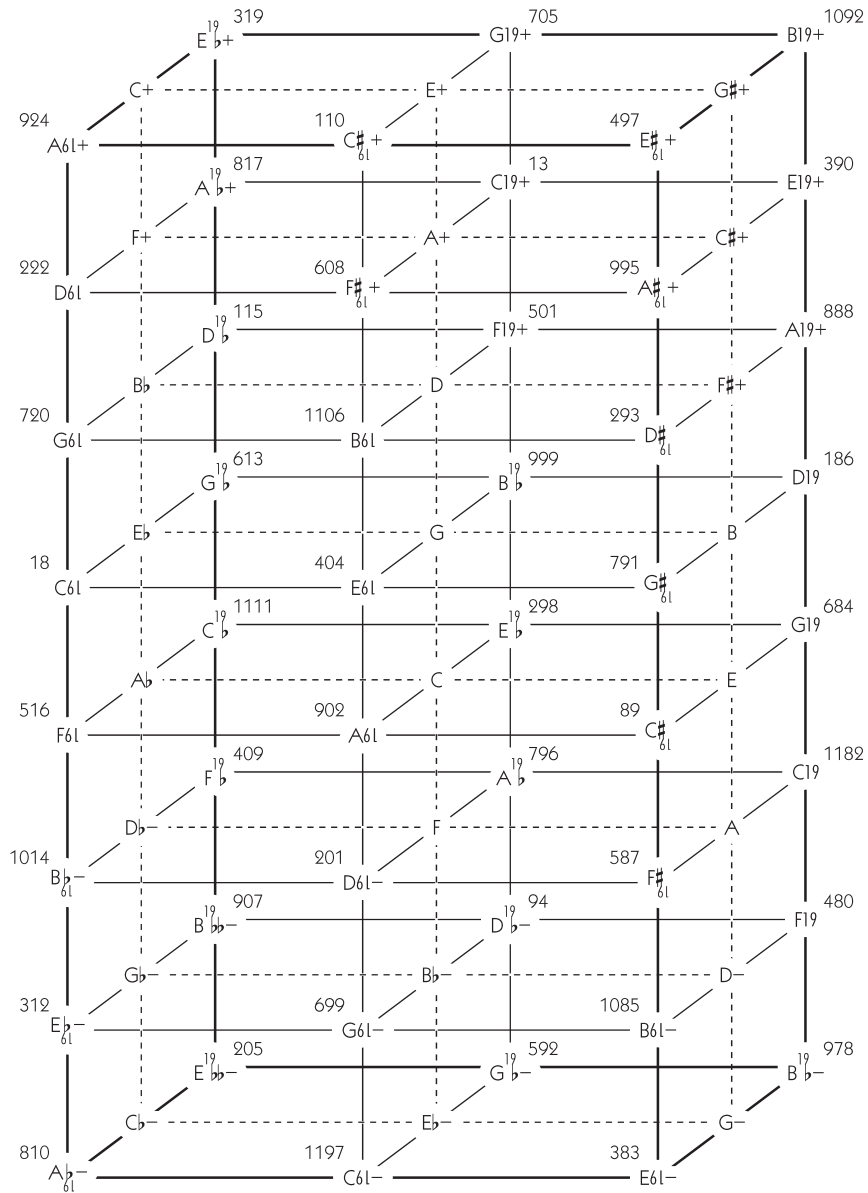
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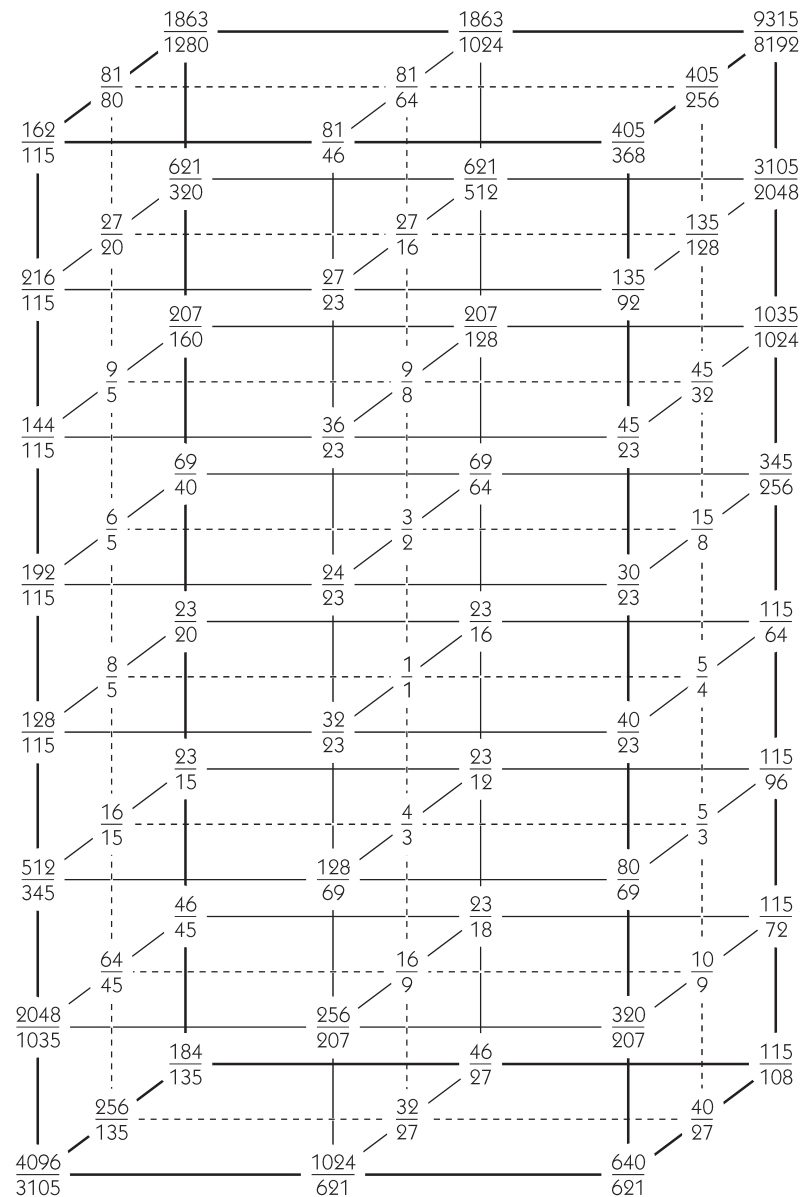
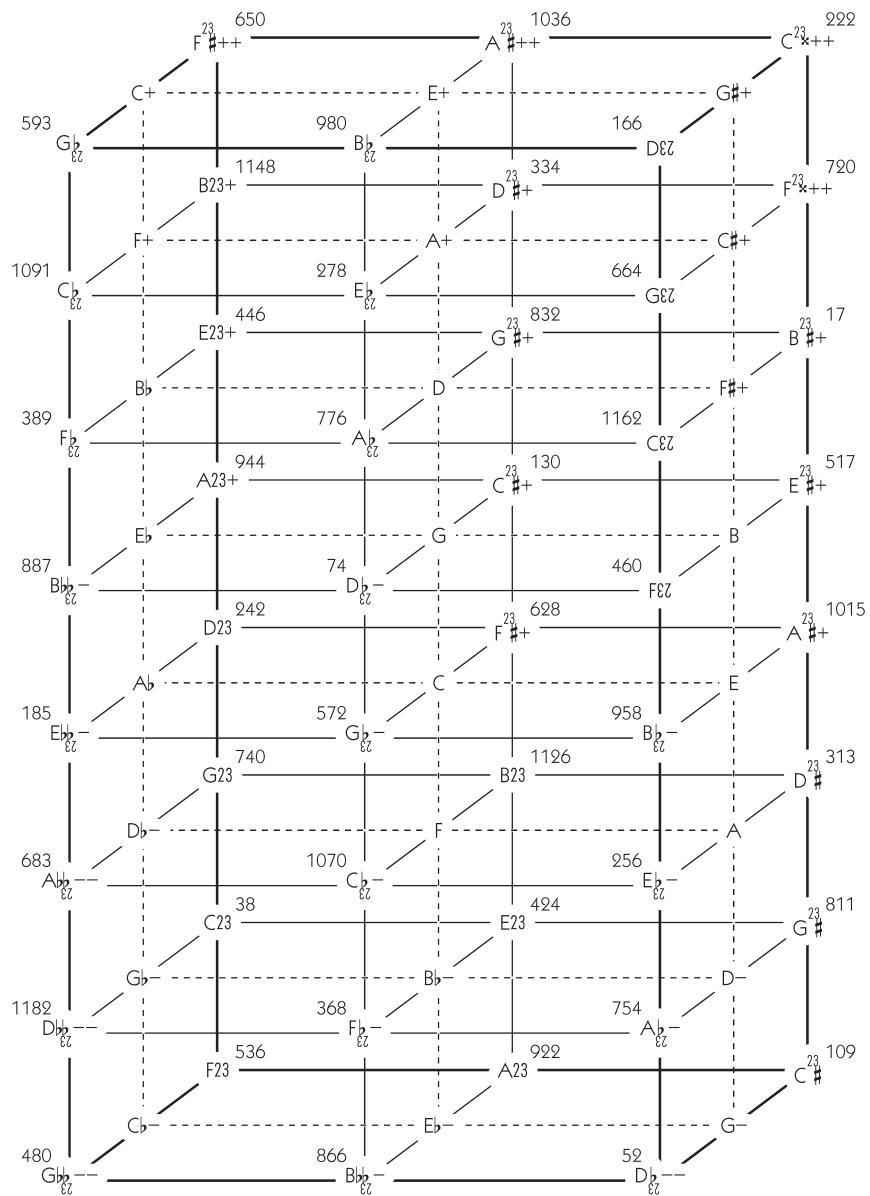
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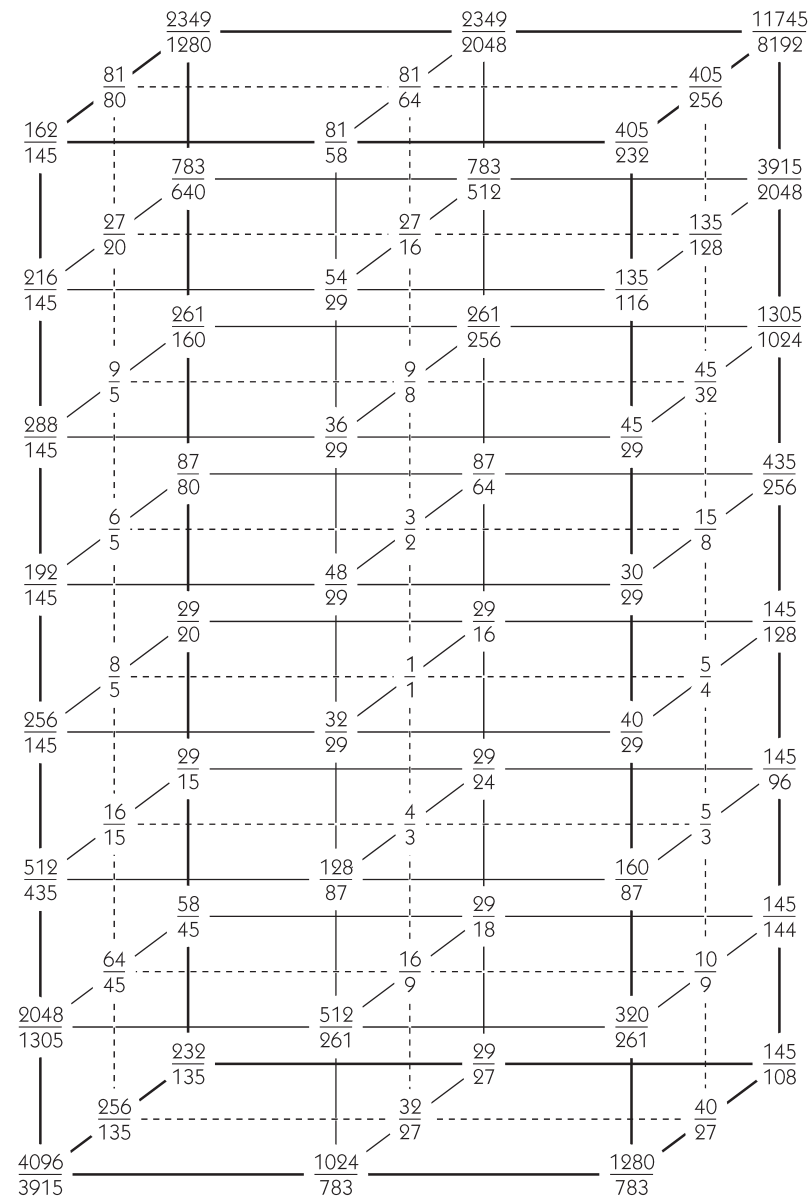
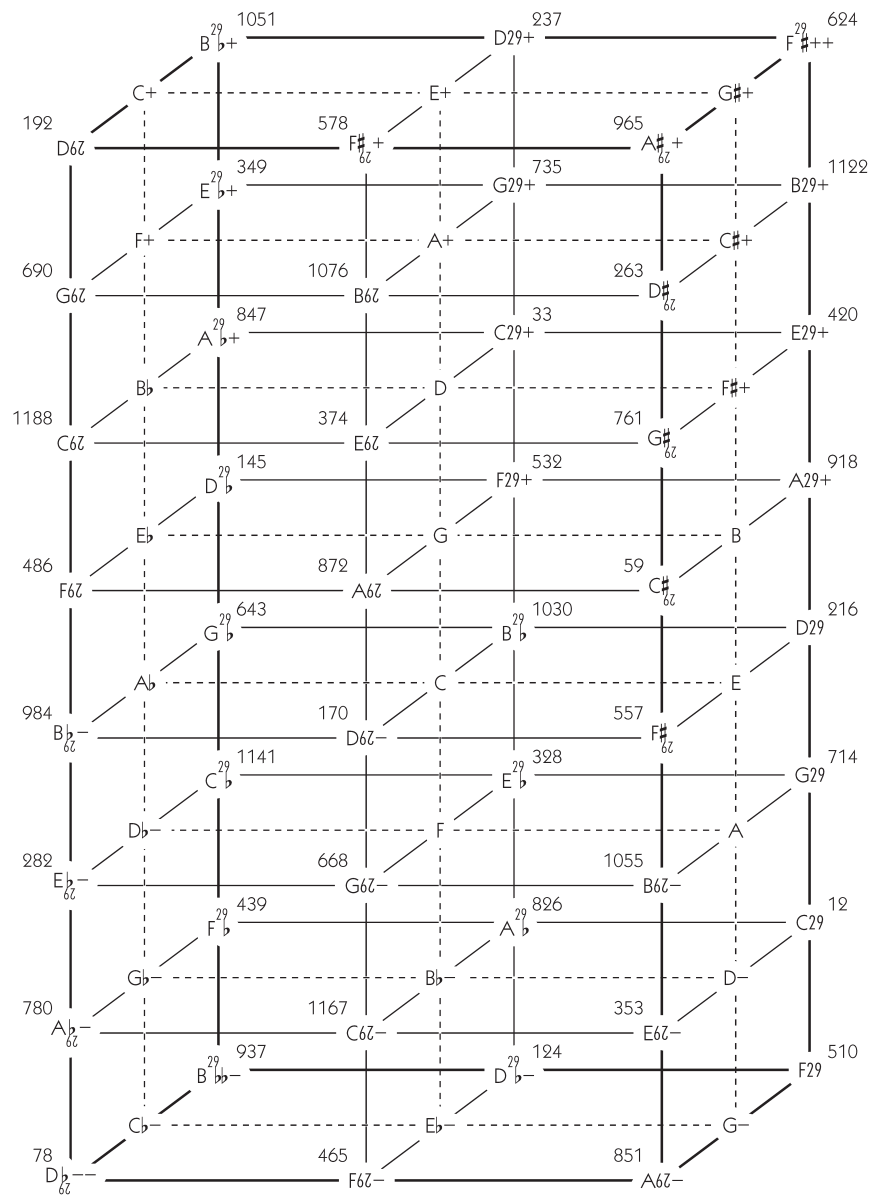
## 2, 3, 5, 19-limit Pitch Lattice (Notation, Ratio and Cent Values)



## 2, 3, 5, 23-limit Pitch Lattice (Notation, Ratio and Cent Values)

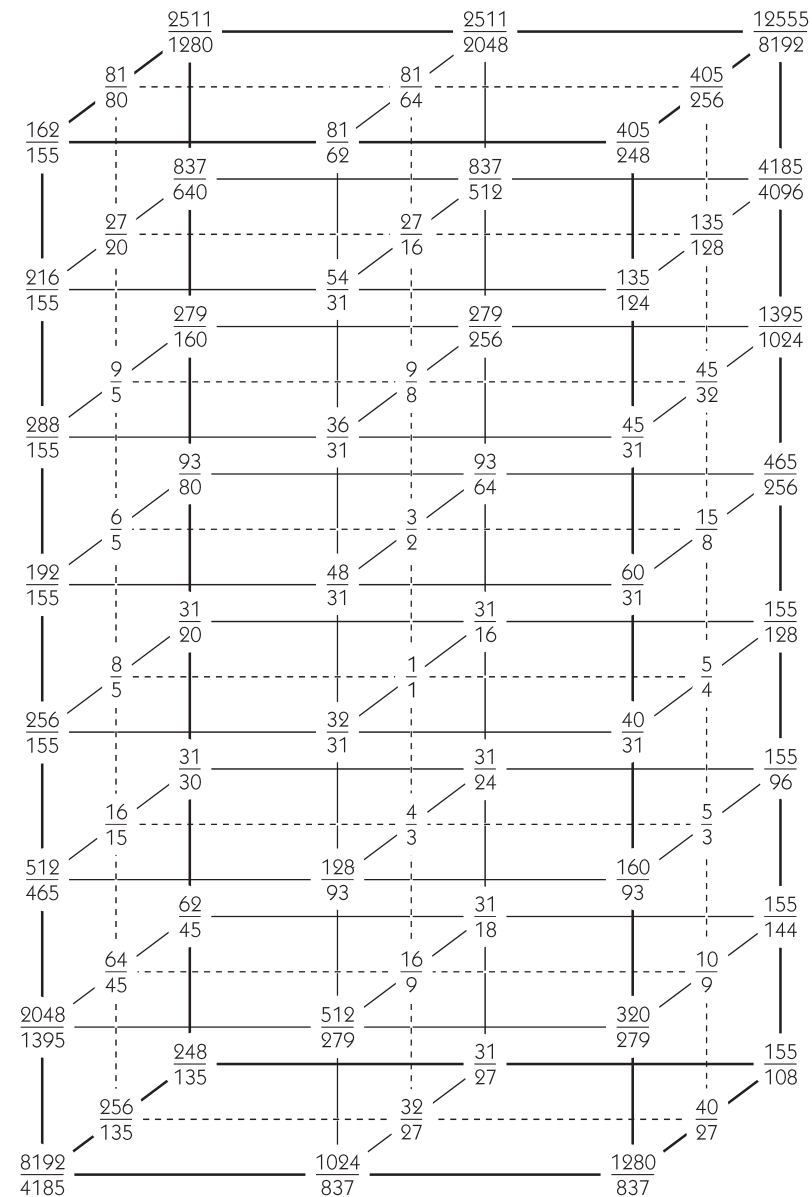
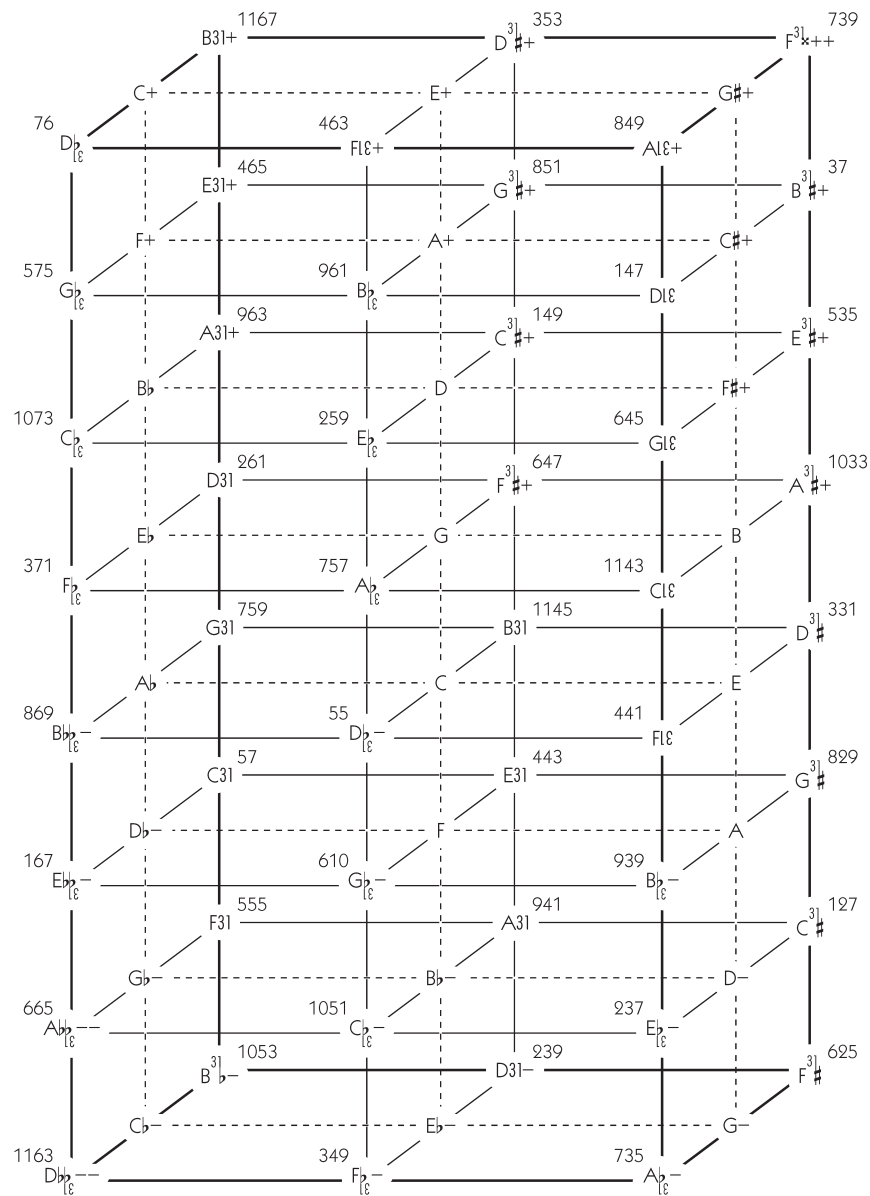


## 2, 3, 5, 29-limit Pitch Lattice (Notation, Ratio and Cent Values)





## 2, 3, 5, 31-limit Pitch Lattice (Notation, Ratio and Cent Values)



## INSTRUMENTATION

Ludisto Unu (Player One)



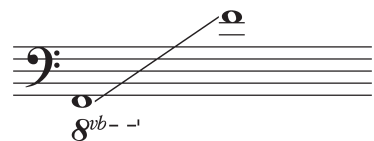
Ludisto Du (Player Two)



Ludisto Tri (Player Three)



Ludisto Kvar (Player Four)



Player 1 to 4 (Sampled '1967 William Dowd French Double Harpsichord' – Akai S3000XL Midi Stereo Digital Sampler and four 61-key Midi Keyboards)

Transposed score

## PERFORMANCE NOTES

In this score, accidentals apply only to the subsequent note in sections utilizing proportional notation, whereby in sections utilizing standard metric notation, accidentals apply throughout the bar.



fermata (approximately 2")



long comma (approximately 1")



short comma (approximately 0.5")

The notation utilized in the score is based on Benjamin Johnston's 'Extended Just Intonation'. It contains 23 unique symbols, which may be combined in any manner – 5 identical to conventional sharp ( $\sharp$ ), flat ( $\flat$ ), double sharp ( $\times$ ), double flat ( $\flat\flat$ ), and natural ( $\natural$ ) accidentals (raising, lowering or neutralizing a tone by  $25/24$  and  $625/576$ , or 70.672 and 141.345 cents). Additional symbols include (+) and (–), raising or lowering a tone by  $81/80$  (one syntonic comma), or 21.506 cents, as well as the following utonal and otonal sets of symbols for each partial chroma up to the thirty-first harmonic: (7) and (L), lowering or raising a tone by  $36/35$  (one septimal comma), or 48.770 cents; ( $\uparrow$ ) and ( $\downarrow$ ), raising or lowering a tone by  $33/32$  (one undecimal comma), or 53.273 cents; (13) and (13), raising or lowering a tone by  $65/64$  (one tridecimal comma), or 26.841 cents; (17) and (L1), raising or lowering a tone by  $51/50$  (one septendecimal comma), or 34.283 cents; (19) and (61), lowering or raising a tone by  $96/95$  (one nonadecimal comma), or 18.128 cents; (23) and (23), raising or lowering a tone by  $46/45$  (one trivigesimal comma), or 38.051 cents; (29) and (67), raising or lowering a tone by  $145/144$  (one nonavigesimal comma), or 11.981 cents; and finally, (31) and (18), raising or lowering a tone by  $31/30$  (one untrigesimal comma), or 56.767 cents.



raised by  $25/24$ , or one small just chromatic semitone (70.672 cents)



lowered by  $25/24$ , or one small just chromatic semitone (70.672 cents)

|     |                                                                                 |
|-----|---------------------------------------------------------------------------------|
| ×   | raised by $25/24$ plus $25/24$ , or $625/576$ (141.345 cents)                   |
| ♭♭  | lowered by $25/24$ minus $25/24$ , or $625/576$ (141.345 cents)                 |
| ♮   | raised or lowered to natural tone                                               |
| +   | raised by $81/80$ , or one syntonic comma (21.506 cents)                        |
| —   | lowered by $81/80$ , or one syntonic comma (21.506 cents)                       |
| ♯+  | raised by $25/24$ plus $81/80$ , or $135/128$ (92.179 cents)                    |
| ♭—  | lowered by $25/24$ minus $81/80$ , or $135/128$ (92.179 cents)                  |
| ♯++ | raised by $25/24$ plus $81/80$ plus $81/80$ , or $2187/2048$ (113.685 cents)    |
| ♭—— | lowered by $25/24$ minus $81/80$ minus $81/80$ , or $2187/2048$ (113.685 cents) |
| 7   | lowered by $36/35$ , or one septimal comma (48.770 cents)                       |
| ℒ   | raised by $36/35$ , or one septimal comma (48.770 cents)                        |
| 7+  | lowered by $36/35$ plus $81/80$ , or $729/700$ (70.277 cents)                   |
| ℒ—  | raised by $36/35$ minus $81/80$ , or $729/700$ (70.277 cents)                   |
| ♭7  | lowered by $25/24$ minus $36/35$ , or $15/14$ (119.443 cents)                   |

|     |                                                                              |
|-----|------------------------------------------------------------------------------|
| #   | raised by $25/24$ plus $36/35$ , or $15/14$ (119.443 cents)                  |
| b+  | lowered by $25/24$ minus $36/35$ plus $81/80$ , or $243/224$ (140.949 cents) |
| #-  | raised by $25/24$ plus $36/35$ minus $81/80$ , or $243/224$ (140.949 cents)  |
| ↑   | raised by $33/32$ , or the 33rd harmonic (53.273 cents)                      |
| ↓   | lowered by $33/32$ , or the 33rd harmonic (53.273 cents)                     |
| ↑-  | raised by $33/32$ minus $81/80$ , or $55/54$ (31.767 cents)                  |
| ↓+  | lowered by $33/32$ plus $81/80$ , or $55/54$ (31.767 cents)                  |
| ↑b  | lowered by $25/24$ plus $33/32$ , or $100/99$ (17.399 cents)                 |
| #↓  | raised by $25/24$ plus $33/32$ , or $100/99$ (17.399 cents)                  |
| ↑b- | lowered by $25/24$ plus $33/32$ minus $81/80$ , or $45/44$ (38.906 cents)    |
| #↑  | raised by $25/24$ minus $33/32$ plus $81/80$ , or $45/44$ (38.906 cents)     |
| 13  | raised by $65/64$ (26.841 cents)                                             |
| ℰl  | lowered by $65/64$ (26.841 cents)                                            |
| 13+ | raised by $65/64$ plus $81/80$ , or $1053/1024$ (48.348 cents)               |

|                       |                                                                           |
|-----------------------|---------------------------------------------------------------------------|
| $\xi l -$             | lowered by $65/64$ minus $81/80$ , or $1053/1024$ (48.348 cents)          |
| $l3 -$                | raised by $65/64$ minus $81/80$ , or $325/324$ (5.335 cents)              |
| $\xi l +$             | lowered by $65/64$ plus $81/80$ , or $325/324$ (5.335 cents)              |
| $l3$<br>$b$           | lowered by $25/24$ plus $65/64$ , or $40/39$ (43.831 cents)               |
| $\sharp$<br>$\xi l$   | raised by $25/24$ minus $65/64$ , or $40/39$ (43.831 cents)               |
| $l3$<br>$b -$         | lowered by $25/24$ plus $65/64$ minus $81/80$ , or $27/26$ (65.337 cents) |
| $\sharp +$<br>$\xi l$ | raised by $25/24$ minus $65/64$ plus $81/80$ , or $27/26$ (65.337 cents)  |
| $l7$                  | raised by $51/50$ (34.283 cents)                                          |
| $Ll$                  | lowered by $51/50$ (34.283 cents)                                         |
| $l7 -$                | raised by $51/50$ minus $81/80$ , or $136/135$ (12.777 cents)             |
| $Ll +$                | lowered by $51/50$ plus $81/80$ , or $136/135$ (12.777 cents)             |
| $l7$<br>$\sharp$      | raised by $25/24$ plus $51/50$ , or $17/16$ (104.955 cents)               |
| $b$<br>$Ll$           | lowered by $25/24$ minus $51/50$ , or $17/16$ (104.955 cents)             |
| $l7$<br>$\sharp -$    | raised by $25/24$ plus $51/50$ minus $81/80$ , or $85/81$ (83.449 cents)  |

$\flat^+$   
6l lowered by  $25/24$  minus  $51/50$  plus  $81/80$ , or  $85/81$  (83.449 cents)

19 lowered by  $96/95$  (18.128 cents)

6l raised by  $96/95$  (18.128 cents)

19+ lowered by  $96/95$  plus  $81/80$ , or  $512/513$  (−3.378 cents)

6l− raised by  $96/95$  minus  $81/80$ , or  $512/513$  (−3.378 cents)

19− lowered by  $96/95$  minus  $81/80$ , or  $486/475$  (39.635 cents)

6l+ raised by  $96/95$  plus  $81/80$ , or  $486/475$  (39.635 cents)

19  
 $\flat$  lowered by  $25/24$  minus  $96/95$ , or  $20/19$  (88.801 cents)

$\sharp$   
6l raised by  $25/24$  plus  $96/95$ , or  $20/19$  (88.801 cents)

19  
 $\flat$ − lowered by  $25/24$  minus  $96/95$  minus  $81/80$ , or  $81/76$  (110.307 cents)

$\sharp$  +  
6l raised by  $25/24$  plus  $96/95$  plus  $81/80$ , or  $81/76$  (110.307 cents)

23 raised by  $46/45$  (38.051 cents)

87 lowered by  $46/45$  (38.051 cents)

23+ raised by  $46/45$  plus  $81/80$ , or  $207/200$  (59.557 cents)

|                     |                                                                               |
|---------------------|-------------------------------------------------------------------------------|
| $\xi\zeta-$         | lowered by $46/45$ minus $81/80$ , or $207/200$ (59.557 cents)                |
| $23-$               | raised by $46/45$ minus $81/80$ , or $736/729$ (16.544 cents)                 |
| $\xi\zeta+$         | lowered by $46/45$ plus $81/80$ , or $736/729$ (16.544 cents)                 |
| $23_{\#}$           | raised by $25/24$ plus $46/45$ , or $115/108$ (108.723 cents)                 |
| $\flat_{\xi\zeta}$  | lowered by $25/24$ minus $46/45$ , or $115/108$ (108.723 cents)               |
| $23_{\#}+$          | raised by $25/24$ plus $46/45$ plus $81/80$ , or $69/64$ (130.229 cents)      |
| $\flat_{\xi\zeta}-$ | lowered by $25/24$ minus $46/45$ minus $81/80$ , or $69/64$ (130.229 cents)   |
| $29$                | raised by $145/144$ (11.981 cents)                                            |
| $6\zeta$            | lowered by $145/144$ (11.981 cents)                                           |
| $29+$               | raised by $145/144$ plus $81/80$ , or $261/256$ (33.487 cents)                |
| $6\zeta-$           | lowered by $145/144$ minus $81/80$ , or $261/256$ (33.487 cents)              |
| $29_{\flat}$        | lowered by $25/24$ plus $145/144$ , or $30/29$ (58.692 cents)                 |
| $\sharp_{6\zeta}$   | raised by $25/24$ minus $145/144$ , or $30/29$ (58.692 cents)                 |
| $29_{\flat}-$       | lowered by $25/24$ plus $145/144$ minus $81/80$ , or $243/232$ (80.198 cents) |



|                  |                                                                                                 |
|------------------|-------------------------------------------------------------------------------------------------|
| $\sharp +$<br>67 | raised by $25/24$ minus $145/144$ plus $81/80$ , or $243/232$ (80.198 cents)                    |
| 31               | raised by $31/30$ (56.767 cents)                                                                |
| 18               | lowered by $31/30$ (56.767 cents)                                                               |
| 31+              | raised by $31/30$ plus $81/80$ , or $837/800$ (78.273 cents)                                    |
| 18-              | lowered by $31/30$ minus $81/80$ , or $837/800$ (78.273 cents)                                  |
| 31-              | raised by $31/30$ minus $81/80$ , or $248/243$ (35.261 cents)                                   |
| 18+              | lowered by $31/30$ plus $81/80$ , or $248/243$ (35.261 cents)                                   |
| 31 $\sharp$      | raised by $25/24$ plus $31/30$ , or $155/144$ (127.439 cents)                                   |
| 18 $\flat$       | lowered by $25/24$ minus $31/30$ , or $155/144$ (127.439 cents)                                 |
| 31 $\sharp +$    | raised by $25/24$ plus $31/30$ plus $81/80$ , or $279/256$ (148.946 cents)                      |
| 18 $\flat -$     | lowered by $25/24$ minus $31/30$ minus $81/80$ , or $279/256$ (148.946 cents)                   |
| 31 $\sharp + +$  | raised by $25/24$ plus $31/30$ plus $81/80$ plus $81/80$ , or $22599/20480$ (170.452 cents)     |
| 18 $\flat - -$   | lowered by $25/24$ minus $31/30$ minus $81/80$ minus $81/80$ , or $22599/20480$ (170.452 cents) |

3-limit tuning matrix no. 1 'key of C' (Program 01)

|       |     |              |     |             |     |     |            |     |             |     |             |     |
|-------|-----|--------------|-----|-------------|-----|-----|------------|-----|-------------|-----|-------------|-----|
| NOTE  | C   | D $\flat$ -- | D   | E $\flat$ – | E+  | F   | --         | G   | A $\flat$ – | A+  | B $\flat$ – | B+  |
| KEY   | C   | C $\sharp$   | D   | D $\sharp$  | E   | F   | F $\sharp$ | G   | G $\sharp$  | A   | A $\sharp$  | B   |
| CENTS | +00 | –10          | +04 | –06         | +08 | –02 | +00        | +02 | –08         | +06 | –04         | +10 |

5-limit tuning matrix no. 1 'key of C' (Program 02)

|       |     |             |     |            |     |     |            |     |            |     |             |     |
|-------|-----|-------------|-----|------------|-----|-----|------------|-----|------------|-----|-------------|-----|
| NOTE  | C   | D $\flat$ – | D   | E $\flat$  | E   | F   | --         | G   | A $\flat$  | A   | B $\flat$ – | B   |
| KEY   | C   | C $\sharp$  | D   | D $\sharp$ | E   | F   | F $\sharp$ | G   | G $\sharp$ | A   | A $\sharp$  | B   |
| CENTS | +00 | +12         | +04 | +16        | –14 | –02 | +00        | +02 | +14        | –16 | –04         | –12 |

5-limit tuning matrix no. 2 'key of C' (Program 03)

|       |     |             |     |            |     |     |            |     |            |     |            |     |
|-------|-----|-------------|-----|------------|-----|-----|------------|-----|------------|-----|------------|-----|
| NOTE  | C   | D $\flat$ – | D   | E $\flat$  | E   | F   | --         | G   | A $\flat$  | A   | B $\flat$  | B   |
| KEY   | C   | C $\sharp$  | D   | D $\sharp$ | E   | F   | F $\sharp$ | G   | G $\sharp$ | A   | A $\sharp$ | B   |
| CENTS | +00 | +12         | +04 | +16        | –14 | –02 | +00        | +02 | +14        | –16 | +18        | –12 |

7-limit tuning matrix no. 1 'key of G' (Program 04)

|       |     |                          |     |            |                 |                   |                          |     |                           |                 |            |                 |
|-------|-----|--------------------------|-----|------------|-----------------|-------------------|--------------------------|-----|---------------------------|-----------------|------------|-----------------|
| NOTE  | C   | C $\sharp$ $\frac{1}{2}$ | D   | E $\flat$  | E $\frac{1}{2}$ | F $\frac{7}{4}$ + | F $\sharp$ $\frac{1}{2}$ | G   | A $\flat$ $\frac{1}{2}$ + | A $\frac{1}{2}$ | B $\flat$  | B $\frac{1}{2}$ |
| KEY   | C   | C $\sharp$               | D   | D $\sharp$ | E               | F                 | F $\sharp$               | G   | G $\sharp$                | A               | A $\sharp$ | B               |
| CENTS | +00 | +19                      | +04 | –33        | +35             | –29               | +17                      | +02 | –14                       | +33             | –31        | +37             |

7-limit tuning matrix no. 2 'key of G' (Program 05)

|       |     |            |     |            |              |            |            |     |            |              |            |     |
|-------|-----|------------|-----|------------|--------------|------------|------------|-----|------------|--------------|------------|-----|
| NOTE  | C   | C $\sharp$ | D   | E $\flat$  | E $\natural$ | F $\sharp$ | F $\sharp$ | G   | A $\flat$  | A $\natural$ | B $\flat$  | B   |
| KEY   | C   | C $\sharp$ | D   | D $\sharp$ | E            | F          | F $\sharp$ | G   | G $\sharp$ | A            | A $\sharp$ | B   |
| CENTS | +00 | +19        | +04 | -33        | +35          | -29        | +17        | +02 | -14        | +33          | -31        | -12 |

11-limit tuning matrix no. 1 'key of G' (Program 06)

|       |     |              |     |            |                |            |              |     |            |                |              |     |
|-------|-----|--------------|-----|------------|----------------|------------|--------------|-----|------------|----------------|--------------|-----|
| NOTE  | C   | C $\uparrow$ | D   | --         | E $\downarrow$ | F $\sharp$ | F $\uparrow$ | G   | --         | A $\downarrow$ | B $\uparrow$ | --  |
| KEY   | C   | C $\sharp$   | D   | D $\sharp$ | E              | F          | F $\sharp$   | G   | G $\sharp$ | A              | A $\sharp$   | B   |
| CENTS | +00 | -47          | +04 | +00        | -45            | +37        | -49          | +02 | +00        | -47            | +49          | +00 |

11-limit tuning matrix no. 2 'key of G' (Program 07)

|       |     |              |     |            |                |            |              |     |            |              |              |     |
|-------|-----|--------------|-----|------------|----------------|------------|--------------|-----|------------|--------------|--------------|-----|
| NOTE  | C   | C $\uparrow$ | D   | --         | E $\downarrow$ | F $\sharp$ | F $\uparrow$ | G   | --         | A $\uparrow$ | B $\uparrow$ | B   |
| KEY   | C   | C $\sharp$   | D   | D $\sharp$ | E              | F          | F $\sharp$   | G   | G $\sharp$ | A            | A $\sharp$   | B   |
| CENTS | +00 | -47          | +04 | +00        | -45            | -29        | -49          | +02 | +00        | +06          | +49          | -12 |

11-limit tuning matrix no. 3 'key of G' (Program 08)

|       |            |            |     |            |           |            |              |     |            |                |                |     |
|-------|------------|------------|-----|------------|-----------|------------|--------------|-----|------------|----------------|----------------|-----|
| NOTE  | C $\sharp$ | --         | D   | --         | E $\flat$ | F $\sharp$ | F $\uparrow$ | G   | --         | A $\downarrow$ | B $\downarrow$ | --  |
| KEY   | C          | C $\sharp$ | D   | D $\sharp$ | E         | F          | F $\sharp$   | G   | G $\sharp$ | A              | A $\sharp$     | B   |
| CENTS | +39        | +00        | +04 | +00        | -31       | +37        | -49          | +02 | +00        | -47            | +35            | +00 |

11-limit tuning matrix no. 4 'key of G' (Program 09)

|       |     |                |     |                |                  |                               |                |     |                |                |                  |     |
|-------|-----|----------------|-----|----------------|------------------|-------------------------------|----------------|-----|----------------|----------------|------------------|-----|
| NOTE  | C   | C <sup>↑</sup> | D   | --             | E <sub>↓</sub> + | F <sup>♯</sup> <sub>↓</sub> + | F <sup>↑</sup> | G   | --             | A <sub>↓</sub> | B <sub>↓</sub> - | --  |
| KEY   | C   | C <sup>♯</sup> | D   | D <sup>♯</sup> | E                | F                             | F <sup>♯</sup> | G   | G <sup>♯</sup> | A              | A <sup>♯</sup>   | B   |
| CENTS | +00 | -47            | +04 | +00            | -45              | +37                           | -49            | +02 | +00            | -33            | +49              | +00 |

13-limit tuning matrix no. 1 'key of D' (Program 10)

|       |                 |                               |     |                              |                  |                   |                               |     |                               |     |                              |                |
|-------|-----------------|-------------------------------|-----|------------------------------|------------------|-------------------|-------------------------------|-----|-------------------------------|-----|------------------------------|----------------|
| NOTE  | C <sup>13</sup> | C <sup>♯</sup> <sub>ℓ</sub> + | D   | E <sup>13</sup> <sub>♭</sub> | E <sub>ℓ</sub> + | F <sup>13</sup> + | F <sup>♯</sup> <sub>ℓ</sub> + | G   | G <sup>♯</sup> <sub>ℓ</sub> + | A+  | B <sup>13</sup> <sub>♭</sub> | B <sub>ℓ</sub> |
| KEY   | C               | C <sup>♯</sup>                | D   | D <sup>♯</sup>               | E                | F                 | F <sup>♯</sup>                | G   | G <sup>♯</sup>                | A   | A <sup>♯</sup>               | B              |
| CENTS | +27             | -35                           | +04 | +42                          | -19              | +46               | -37                           | +02 | -33                           | +06 | +44                          | -39            |

13-limit tuning matrix no. 2 'key of D' (Program 11)

|       |                 |                               |     |                              |                  |                   |                               |     |                              |     |                              |                |
|-------|-----------------|-------------------------------|-----|------------------------------|------------------|-------------------|-------------------------------|-----|------------------------------|-----|------------------------------|----------------|
| NOTE  | C <sup>13</sup> | C <sup>♯</sup> <sub>ℓ</sub> + | D   | E <sup>13</sup> <sub>♭</sub> | E <sub>ℓ</sub> + | F <sup>13</sup> + | F <sup>♯</sup> <sub>ℓ</sub> + | G   | A <sup>13</sup> <sub>♭</sub> | A+  | B <sup>13</sup> <sub>♭</sub> | B <sub>ℓ</sub> |
| KEY   | C               | C <sup>♯</sup>                | D   | D <sup>♯</sup>               | E                | F                 | F <sup>♯</sup>                | G   | G <sup>♯</sup>               | A   | A <sup>♯</sup>               | B              |
| CENTS | +27             | -35                           | +04 | +42                          | -19              | +46               | -37                           | +02 | +41                          | +06 | +44                          | -39            |

17-limit tuning matrix no. 1 'key of D' (Program 12)

|       |                  |                              |     |                             |                 |                  |                              |     |                              |     |                             |                 |
|-------|------------------|------------------------------|-----|-----------------------------|-----------------|------------------|------------------------------|-----|------------------------------|-----|-----------------------------|-----------------|
| NOTE  | C <sub>ℓ</sub> + | C <sup>17</sup> <sub>♯</sub> | D   | E <sub>ℓ</sub> <sup>+</sup> | E <sup>17</sup> | F <sub>ℓ</sub> + | F <sup>17</sup> <sub>♯</sub> | G   | G <sup>17</sup> <sub>♯</sub> | A+  | B <sub>ℓ</sub> <sup>+</sup> | B <sup>17</sup> |
| KEY   | C                | C <sup>♯</sup>               | D   | D <sup>♯</sup>              | E               | F                | F <sup>♯</sup>               | G   | G <sup>♯</sup>               | A   | A <sup>♯</sup>              | B               |
| CENTS | -13              | +05                          | +04 | +03                         | +21             | -15              | +03                          | +02 | +07                          | +06 | +05                         | +23             |

17-limit tuning matrix no. 2 'key of D' (Program 13)

|       |              |                         |     |                           |     |              |                           |     |                           |     |                           |     |
|-------|--------------|-------------------------|-----|---------------------------|-----|--------------|---------------------------|-----|---------------------------|-----|---------------------------|-----|
| NOTE  | C $\flat$ 1+ | D $\flat$ $\frac{1}{1}$ | D   | E $\flat$ $\frac{1}{1}$ + | E17 | F $\flat$ 1+ | F $\sharp$ $\frac{17}{1}$ | G   | A $\flat$ $\frac{1}{1}$ + | A+  | B $\flat$ $\frac{1}{1}$ + | B17 |
| KEY   | C            | C $\sharp$              | D   | D $\sharp$                | E   | F            | F $\sharp$                | G   | G $\sharp$                | A   | A $\sharp$                | B   |
| CENTS | -13          | -01                     | +04 | +03                       | +21 | -15          | +03                       | +02 | +01                       | +06 | +05                       | +23 |

19-limit tuning matrix no. 1 'key of A' (Program 14)

|       |      |                              |     |                              |     |      |                              |      |                              |     |                           |     |
|-------|------|------------------------------|-----|------------------------------|-----|------|------------------------------|------|------------------------------|-----|---------------------------|-----|
| NOTE  | C19+ | C $\sharp$ $\frac{19}{61}$ + | D   | D $\sharp$ $\frac{19}{61}$ + | E+  | F19+ | F $\sharp$ $\frac{19}{61}$ + | G19+ | G $\sharp$ $\frac{19}{61}$ + | A+  | B $\flat$ $\frac{19}{61}$ | B61 |
| KEY   | C    | C $\sharp$                   | D   | D $\sharp$                   | E   | F    | F $\sharp$                   | G    | G $\sharp$                   | A   | A $\sharp$                | B   |
| CENTS | +13  | +10                          | +04 | +14                          | +08 | +01  | +08                          | +05  | +12                          | +06 | -01                       | +06 |

23-limit tuning matrix no. 1 'key of A' (Program 15)

|       |               |                              |     |                              |     |               |                              |              |                              |     |                           |      |
|-------|---------------|------------------------------|-----|------------------------------|-----|---------------|------------------------------|--------------|------------------------------|-----|---------------------------|------|
| NOTE  | C $\flat$ 23+ | C $\sharp$ $\frac{23}{87}$ + | D   | D $\sharp$ $\frac{23}{87}$ + | E+  | F $\flat$ 23+ | F $\sharp$ $\frac{23}{87}$ + | G $\flat$ 23 | G $\sharp$ $\frac{23}{87}$ + | A+  | B $\flat$ $\frac{23}{87}$ | B23+ |
| KEY   | C             | C $\sharp$                   | D   | D $\sharp$                   | E   | F             | F $\sharp$                   | G            | G $\sharp$                   | A   | A $\sharp$                | B    |
| CENTS | -17           | +30                          | +04 | +34                          | +08 | -18           | +28                          | -36          | +32                          | +06 | -20                       | +48  |

23-limit tuning matrix no. 2 'key of A' (Program 16)

|       |               |                              |     |                              |     |               |                              |               |                              |     |                           |      |
|-------|---------------|------------------------------|-----|------------------------------|-----|---------------|------------------------------|---------------|------------------------------|-----|---------------------------|------|
| NOTE  | C $\flat$ 23+ | C $\sharp$ $\frac{23}{87}$ + | D   | D $\sharp$ $\frac{23}{87}$ + | E+  | F $\flat$ 23+ | F $\sharp$ $\frac{23}{87}$ + | G $\flat$ 23+ | G $\sharp$ $\frac{23}{87}$ + | A+  | B $\flat$ $\frac{23}{87}$ | B23+ |
| KEY   | C             | C $\sharp$                   | D   | D $\sharp$                   | E   | F             | F $\sharp$                   | G             | G $\sharp$                   | A   | A $\sharp$                | B    |
| CENTS | -17           | +30                          | +04 | +34                          | +08 | -18           | +28                          | -15           | +32                          | +06 | -20                       | +48  |

29-limit tuning matrix no. 1 'key of E' (Program 17)

|       |      |                               |      |                               |     |      |                               |      |                               |     |                               |     |
|-------|------|-------------------------------|------|-------------------------------|-----|------|-------------------------------|------|-------------------------------|-----|-------------------------------|-----|
| NOTE  | C29+ | $C_{\frac{29}{62}}^{\sharp}+$ | D29+ | $D_{\frac{29}{62}}^{\sharp}+$ | E+  | F29+ | $F_{\frac{29}{62}}^{\sharp}+$ | G29+ | $G_{\frac{29}{62}}^{\sharp}+$ | A+  | $A_{\frac{29}{62}}^{\sharp}+$ | B+  |
| KEY   | C    | C $\sharp$                    | D    | D $\sharp$                    | E   | F    | F $\sharp$                    | G    | G $\sharp$                    | A   | A $\sharp$                    | B   |
| CENTS | +33  | -20                           | +37  | -16                           | +08 | +32  | -22                           | +35  | -18                           | +06 | -35                           | +10 |

31-limit tuning matrix no. 1 'key of E' (Program 18)

|       |      |                               |      |                               |     |      |                               |      |                               |     |            |     |
|-------|------|-------------------------------|------|-------------------------------|-----|------|-------------------------------|------|-------------------------------|-----|------------|-----|
| NOTE  | C18+ | $C_{\frac{31}{18}}^{\sharp}+$ | D18+ | $D_{\frac{31}{18}}^{\sharp}+$ | E+  | F18+ | $F_{\frac{31}{18}}^{\sharp}+$ | G18+ | $G_{\frac{31}{18}}^{\sharp}+$ | A+  | --         | B+  |
| KEY   | C    | C $\sharp$                    | D    | D $\sharp$                    | E   | F    | F $\sharp$                    | G    | G $\sharp$                    | A   | A $\sharp$ | B   |
| CENTS | -35  | +49                           | -31  | +53                           | +08 | -37  | +47                           | -33  | +51                           | +06 | +00        | +10 |

31-limit tuning matrix no. 2 'key of E' (Program 19)

|       |      |                               |      |                               |     |      |                               |      |                               |     |            |                                |
|-------|------|-------------------------------|------|-------------------------------|-----|------|-------------------------------|------|-------------------------------|-----|------------|--------------------------------|
| NOTE  | C18+ | $C_{\frac{31}{18}}^{\sharp}+$ | D18+ | $D_{\frac{31}{18}}^{\sharp}+$ | E+  | F18+ | $F_{\frac{31}{18}}^{\sharp}+$ | G18+ | $G_{\frac{31}{18}}^{\sharp}+$ | A+  | --         | $A_{\frac{31}{18}}^{\sharp}++$ |
| KEY   | C    | C $\sharp$                    | D    | D $\sharp$                    | E   | F    | F $\sharp$                    | G    | G $\sharp$                    | A   | A $\sharp$ | B                              |
| CENTS | -35  | +49                           | -31  | +53                           | +08 | -37  | +47                           | -33  | +51                           | +06 | +00        | -45                            |

## Ben Johnston's Fifty-Three-Tone Just Division of the Octave

Relative Pitch:  $A_4 = 440\text{Hz}$  /  $C_4$  (middle C) =  $261.6255654\text{Hz}$

| DEGREE<br>NUMBER | NOTE             | INTERVAL                                                   | RATIO<br>(FRACTION) | RATIO<br>(DECIMAL) | FREQUENCY<br>(HERTZ) | CENTS   | TUNING |
|------------------|------------------|------------------------------------------------------------|---------------------|--------------------|----------------------|---------|--------|
| 01               | C                | unison                                                     | 1/1                 | 1.000000           | 261.626              | 0.000   | +00    |
| 02               | C+               | syntonic comma                                             | 81/80               | 1.012500           | 264.896              | 21.506  | +22    |
| 03               | D $\flat\flat$ − | diminished second, or great diesis                         | 128/125             | 1.024000           | 267.905              | 41.059  | +41    |
| 04               | C $\sharp$       | grave or small just chromatic semitone, or minor half-tone | 25/24               | 1.041667           | 272.527              | 70.672  | −29    |
| 05               | C $\sharp$ +     | Pythagorean acute or large limma                           | 135/128             | 1.054688           | 275.933              | 92.179  | −08    |
| 06               | D $\flat$ −      | just diatonic semitone, or major half-tone                 | 16/15               | 1.066667           | 279.067              | 111.731 | +12    |
| 07               | D $\flat$        | great limma, acute or large half-tone                      | 27/25               | 1.080000           | 282.556              | 133.238 | +33    |
| 08               | C $\times$ +     | acute or large double augmented octave                     | 1125/1024           | 1.098633           | 287.430              | 162.851 | −37    |
| 09               | D−               | just minor tone                                            | 10/9                | 1.111111           | 290.695              | 182.404 | −18    |
| 10               | D                | just major tone (9th harmonic)                             | 9/8                 | 1.125000           | 294.329              | 203.910 | +04    |
| 11               | D+               | acute or large tone                                        | 729/640             | 1.139063           | 298.008              | 225.416 | +25    |
| 12               | E $\flat\flat$   | diminished third                                           | 144/125             | 1.152000           | 301.393              | 244.969 | +45    |
| 13               | D $\sharp$       | augmented second (75th harmonic)                           | 75/64               | 1.171875           | 306.592              | 274.582 | −25    |
| 14               | E $\flat$ −      | Pythagorean minor third, or trihemitone                    | 32/27               | 1.185185           | 310.075              | 294.135 | −06    |
| 15               | E $\flat$        | just minor third                                           | 6/5                 | 1.200000           | 313.951              | 315.641 | +16    |
| 16               | D $\times$       | double augmented second                                    | 625/512             | 1.220703           | 319.367              | 345.255 | +45    |
| 17               | E−               | grave or small major third                                 | 100/81              | 1.234568           | 322.995              | 364.807 | −35    |
| 18               | E                | just major third (5th harmonic)                            | 5/4                 | 1.250000           | 327.032              | 386.314 | −14    |
| 19               | E+               | Pythagorean major third, or ditone (81st harmonic)         | 81/64               | 1.265625           | 331.120              | 407.820 | +08    |
| 20               | F $\flat$        | diminished fourth                                          | 32/25               | 1.280000           | 334.881              | 427.373 | +27    |
| 21               | E $\sharp$       | augmented third                                            | 125/96              | 1.302083           | 340.658              | 456.986 | −43    |
| 22               | F−               | grave or small fourth                                      | 320/243             | 1.316872           | 344.527              | 476.539 | −23    |
| 23               | F                | just and Pythagorean perfect fourth                        | 4/3                 | 1.333333           | 348.834              | 498.045 | −02    |
| 24               | F+               | acute or large fourth                                      | 27/20               | 1.350000           | 353.195              | 519.551 | +20    |
| 25               | G $\flat\flat$ − | grave or small double diminished fifth                     | 512/375             | 1.365333           | 357.206              | 539.104 | +39    |
| 26               | F $\sharp$       | grave or small augmented fourth                            | 25/18               | 1.388889           | 363.369              | 568.717 | −31    |

| DEGREE<br>NUMBER | NOTE             | INTERVAL                                          | RATIO<br>(FRACTION) | RATIO<br>(DECIMAL) | FREQUENCY<br>(HERTZ) | CENTS    | TUNING |
|------------------|------------------|---------------------------------------------------|---------------------|--------------------|----------------------|----------|--------|
| 27               | F $\sharp$ +     | just tritone, or augmented fourth (45th harmonic) | 45/32               | 1.406250           | 367.911              | 590.224  | −10    |
| 28               | G $\flat$ −      | diminished fifth                                  | 64/45               | 1.422222           | 372.090              | 609.776  | +10    |
| 29               | G $\flat$        | acute or large diminished fifth                   | 36/25               | 1.440000           | 376.741              | 631.283  | +31    |
| 30               | F $\times$ +     | acute or large double augmented fourth            | 375/256             | 1.464844           | 383.241              | 660.896  | −39    |
| 31               | G−               | grave or small fifth                              | 40/27               | 1.481481           | 387.593              | 680.449  | −20    |
| 32               | G                | just and Pythagorean perfect fifth (3rd harmonic) | 3/2                 | 1.500000           | 392.438              | 701.955  | +02    |
| 33               | G+               | acute or large fifth                              | 243/160             | 1.518750           | 397.344              | 723.014  | +23    |
| 34               | A $\flat\flat$   | diminished sixth                                  | 192/125             | 1.536000           | 401.857              | 743.014  | +43    |
| 35               | G $\sharp$       | augmented fifth (25th harmonic)                   | 25/16               | 1.562500           | 408.790              | 772.627  | −27    |
| 36               | A $\flat$ −      | Pythagorean minor sixth                           | 128/81              | 1.580247           | 413.433              | 792.180  | −08    |
| 37               | A $\flat$        | just minor sixth                                  | 8/5                 | 1.600000           | 418.601              | 813.686  | +14    |
| 38               | G $\times$       | double augmented fifth                            | 625/384             | 1.627604           | 425.823              | 843.300  | +43    |
| 39               | A−               | grave or small major sixth                        | 400/243             | 1.646091           | 430.659              | 862.852  | −37    |
| 40               | A                | just major sixth                                  | 5/3                 | 1.666667           | 436.043              | 884.359  | −16    |
| 41               | A+               | Pythagorean major sixth (27th harmonic)           | 27/16               | 1.687500           | 441.493              | 905.865  | +06    |
| 42               | B $\flat\flat$ − | diminished seventh                                | 128/75              | 1.706667           | 446.508              | 925.418  | +25    |
| 43               | A $\sharp$       | augmented sixth                                   | 125/72              | 1.736111           | 454.211              | 955.031  | −45    |
| 44               | A $\sharp$ +     | acute or large augmented sixth                    | 225/128             | 1.757813           | 459.889              | 976.537  | −23    |
| 45               | B $\flat$ −      | Pythagorean minor seventh                         | 16/9                | 1.777778           | 465.112              | 996.090  | −04    |
| 46               | B $\flat$        | acute or large minor seventh                      | 9/5                 | 1.800000           | 470.926              | 1017.596 | +18    |
| 47               | A $\times$ +     | acute or large double augmented sixth             | 1875/1024           | 1.831055           | 479.051              | 1047.210 | +47    |
| 48               | B−               | grave or small major seventh                      | 50/27               | 1.851852           | 484.492              | 1066.762 | −33    |
| 49               | B                | just diatonic major seventh (15th harmonic)       | 15/8                | 1.875000           | 490.548              | 1088.269 | −12    |
| 50               | B+               | Pythagorean major seventh                         | 243/128             | 1.898438           | 496.680              | 1109.775 | +10    |
| 51               | C $\flat$        | diminished octave                                 | 48/25               | 1.920000           | 502.321              | 1129.328 | +29    |
| 52               | B $\sharp$       | meantone augmented seventh (125th harmonic)       | 125/64              | 1.953125           | 510.987              | 1158.941 | −41    |
| 53               | C−               | grave or small octave                             | 160/81              | 1.975309           | 516.791              | 1178.494 | −22    |
| 01               | C                | octave                                            | 2/1                 | 2.000000           | 523.251              | 1200.000 | +00    |



Hommage à Lou Harrison (1917-2003)

# La Homa Kanto

for Harmonically Tuned Synthesizer Quartet

This whole round living world – the Human Music – rouses &  
– delights me, it stirs me to a “trans-ethnic”, a planetary music.  
*Lou Harrison's Music Primer, 1971*

Andrián Pertout, No. 393  
2005

(A) 3-limit intonation

MIDI 2 [PG 01] *senza tempo*

Ludisto Unu

MIDI 3 [PG 01]

Ludisto Du

MIDI 4 [PG 01]

Ludisto Tri

*p > pp*

MIDI 5 [PG 01]

Ludisto Kvar

*p > pp*

10"

$\text{♩} = 60 \text{ } 1+3+1$

*mp > p*

*rit.*

*p < mp > p*

*senza tempo*

Unu

*p > pp*

Du

*p > pp*

Tri

*p > pp*

Kvar

*p > pp*

10"

$\text{♩} = 60$

*mp > p*

*molto rit.*

*p < mp > p*

*p < mp > p*

*mp > p*

[illegible]

**B** 5-limit intonation  
*senza tempo*

Unu

Du

Tri

Kvar

PG 02

$p > pp$

10"

$\text{♩} = 60$

PG 02

$p$

PG 02

$p < mp > p$

*rit.*

$mp$

Unu

Du

Tri

Kvar

$pp$

$pp$

$p > pp$

$p > pp$

10"

$\text{♩} = 60$

$mp > p$

$mp > p$

*molto rit.*

$p < mp > p$

$p < mp > p$

senza tempo

10"

Unu

Du

Tri

Kvar

PG 03

*mp* > *p*

*mf* > *mp*

*rit.*

*mp* < *mf* > *mp*

*mf*

*mf*

$\text{♩} = 60$

senza tempo

10"

Unu

Du

Tri

Kvar

PG 02

*p*

*p*

*p*

*mp* > *p*

$\text{♩} = 60$

*mf* > *mp*

*molto rit.*

*mf*

*mp* < *mf*

*mp* < *mf*

*mf* > *mp*

*mf*

AP • 393

senza tempo

Unu *mp* > *p*

Du *p* *mp* > *p*

Tri *mp* > *p*

Kvar > *pp*

12"

$\text{♩} = 60$

*mf* *mp* (sub)

*mf* *mp* (sub)

*mp* *p* (sub)

*mp* *p* (sub)

*mf*

*mf* > *mp*

*rit.*

45

Unu *p* *mp* > *p* *molto rit.* *mf* > *mp*

Du *p* *mf* > *mp* *mf* > *mp*

Tri *p* *mf* > *mp* *mf*

Kvar *mp* > *p* *mp* < *mf* > *mp*

53

PG 08

PG 06

molto rit.

PG 08

PG 06

PG 08

PG 06

pp

pp

pp

pp

mp > p

mp > p

mp > p

mp > p

mp < mf > mp

mp < mf > mp

mp > p

mf > mp

mp > p

mp

The Song of the Lark

[illegible]



**E** 13-limit intonation

65 PG 10 ♩. = 84

Unu *ppp* *mp* *p*

Du *p* *pp* *ppp* *p*

Tri *p* *pp* *mp* *p*

Kvar *p* *pp* *p* *mp* *p*

69

Unu *pp* *p* *pp* *mp* *p* *mp* *p*

Du *p* *pp* *p* *pp* *mp* *p* *mp* *p*

Tri *pp* *mp* *p* *mp* *p*

Kvar *p* *pp* *p* *pp* *mp* *p* *mp* *p*

73

Unu *ppp* *mf* *mp*

Du *mp* *p* *ppp* *mf* *mp*

Tri *mp* *p* *mf* *mp*

Kvar *mp* *p* *mp* *mf* *mp*

PG 11

77

Unu *mp* *p* *mf* *mp*

Du *mp* *p* *mf* *mp*

Tri *pp* *mf* *mp* *mp* *mf* *mp*

Kvar *mp* *p* *pp* *mp* *mf* *mp*

**F** 17-limit intonation

81 PG 12

Unu *pp* *mp* *p*

Du *p* *pp* *pp* *pp*

Tri *p* *pp* *mp* *p*

Kvar *p* *pp* *p* *mp* *p*

85

Unu *pp* *p* *pp* *mp* *p*

Du *p* *pp* *p* *pp* *mp* *p*

Tri *mp* *p* *mp* *p*

Kvar *pp* *p* *pp* *p*

89

Unu *pp* *PG 13* *mf*  $\rightrightarrows$  *mp*

Du *mp*  $\rightrightarrows$  *p* *pp* *mf*  $\rightrightarrows$  *mp*

Tri *mp*  $\rightrightarrows$  *p* *mf*  $\rightrightarrows$  *mp*

Kvar *mp*  $\rightrightarrows$  *p* *mp*  $\rightrightarrows$  *mf*  $\rightrightarrows$  *mp*

93

Unu *p* *mp*  $\rightrightarrows$  *p* *mf*  $\rightrightarrows$  *mp* *rit.* *mf*

Du *mp*  $\rightrightarrows$  *p* *mp*  $\rightrightarrows$  *p* *mf*  $\rightrightarrows$  *mp* *mf*

Tri *p* *mp*  $\rightrightarrows$  *p* *mf*  $\rightrightarrows$  *mp* *mf*

Kvar *mp*  $\rightrightarrows$  *p* *mp*  $\rightrightarrows$  *p* *mf*  $\rightrightarrows$  *mp* *mf*

$\text{♩} = 60$

**G** 19-limit intonation  
senza tempo

Unu

Du

Tri

Kvar

PG 14

PG 14

PG 14

PG 14

10"

$\text{♩} = 60 \quad 1+3+1$

*mp* *p*

*p* *mp* *p*

*p* *pp*

*pp*

*rit.*



senza tempo

10"

$\bullet = 60$

*mp* > *p*

*mp* > *mf* > *mp*

*rit.*

*mf* > *mp*

Unu

Du

Tri

Kvar

senza tempo

10"

$\bullet = 60$

*mp* > *p*

*mp* > *mf* > *mp*

*molto rit.*

*mp* < *mf* > *mp*

*mf* > *mp*

Unu

Du

Tri

Kvar

Unu

Du

Tri

Kvar

10"

*pp*

*pp*

*p* > *pp*

*p* > *pp*

*mp* > *p*

*molto rit.*

*p* < *mp* > *p*

*p* < *mp* > *p*

*mp* > *p*

senza tempo

Unu

Du

Tri

Kvar

PG 16

$mp > p$

10"

$\text{♩} = 60$

$mf > mp$

$mp > p$

$mp$

$mp < mf$

$mf > mp$

$mf$

senza tempo

Unu

Du

Tri

Kvar

PG 16

$mp > p$

$mp > p$

$mp > p$

$mp > p$

10"

$\text{♩} = 60$

$mf > mp$

$mf$

$mp < mf$

$mp < mf$

$mf > mp$

$mf$

*molto rit.*



**I** 29-limit intonation

♩ = 84

129

PG 17

Unu

Du

Tri

Kvar

*ppp*

*mp*  $\triangleright$  *p*

*p*  $\triangleright$  *pp*

*ppp*

*mp*  $\triangleright$  *p*

*p*  $\triangleright$  *pp*

*p*  $\triangleright$  *pp*

*p*  $\triangleleft$  *mp*  $\triangleright$  *p*

133

Unu

Du

Tri

Kvar

*pp*

*p*  $\triangleright$  *pp*

*pp*

*mp*  $\triangleright$  *p*

*p*  $\triangleright$  *pp*

*pp*

*mp*  $\triangleright$  *p*

*pp*

*mp*  $\triangleright$  *p*

*pp*

*p*  $\triangleright$  *pp*

*p*  $\triangleright$  *pp*

136

Unu *mp* *p* *ppp*

Du *mp* *p* *mp* *p*<sup>2</sup>

Tri *mp* *p* *mp* *p*

Kvar *mp* *p* *mp* *p*

139

Unu *mf* *mp* *mp* *mf* *mp* *p*

Du *ppp* *mp* *p* *pp* *p*

Tri *mf* *mp* *pp* *p*

Kvar *mp* *mf* *mp* *p*

**J** 31-limit intonation

145 PG 18

Unu *pp* *mp* *p*

Du *p* *pp* *p* *pp*

Tri *p* *pp* *mp* *p*

Kvar *p* *pp* *p* *p*

151

PG 18

Unu

Du

Tri

Kvar

*mp* *p*

*mp* *p*

*pp*

*mp* *p*

*mp* *p*

*mp* *p*

*p*

154

Unu

Du

Tri

Kvar

*mf*  $\rightrightarrows$  *mp*

*mp*  $\rightrightarrows$  *p*

*pp*

*mf*  $\rightrightarrows$  *mp*

*mp*  $\rightrightarrows$  *p*

*mp*  $\rightrightarrows$  *mf*  $\rightrightarrows$  *mp*

157

Unu

Du

Tri

Kvar

*p*

*mp*  $\rightrightarrows$  *p*

*mp*  $\rightrightarrows$  *p*

*mf*  $\rightrightarrows$  *mp*

*p*

*mf*  $\rightrightarrows$  *mp*

*mp*  $\rightrightarrows$  *p*

*mp*  $\rightrightarrows$  *p*

160

Unu *mf*  $\rightrightarrows$  *mp*

Du *mf*  $\rightrightarrows$  *mp*

Tri *mf*  $\rightrightarrows$  *mp*

Kvar *mf*  $\rightrightarrows$  *mp*

163

rit.  $\rightarrow$   $\text{♩} = 60$

Unu *mf*  $\rightrightarrows$  *mp*

Du *mf*  $\rightrightarrows$  *mp*

Tri *mf*  $\rightrightarrows$  *mp*

Kvar *mp*